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THE FUNDAMENTAL LAWS OF ARITHMETIC.[1]

[INTRODUCTORY NOTE.

Friedrich Ludwig Gottlob Frege was born on November 8, 1848, at Wismar, and since 1874 has taught at the University of Jena. It is well known that there has been in mathematics, especially during the last century, a constantly growing tendency toward more rigorous proofs and a more accurate determination of the limits of validity of mathematical propositions. For these purposes accurate definitions of mathematical concepts were needed; thus we obtained a much greater distinctness in definitions of a function, of the limits and continuity of a function, of the infinite, and of negative and irrational numbers. A natural continuation of this path of research led to the investigation of the question whether the concept of whole number is capable of definition, and whether the simplest laws which hold for integers are capable of proof. This we see in the work of Cantor, Dedekind, and Frege.[2] The object of a proof, as Frege has said, is not merely to raise the truth of a proposition above every doubt, but also to impart an insight into the dependence of truths on one another. The farther these investigations are continued, the fewer will be the fundamental truths to which everything can be reduced; and this simplification is in itself an end worthy to be striven for.

It was this desire for simplification, together with the philosophical questions as to the *a priori* or *a posteriori*, synthetic or analytic, nature of arithmetical truths, which moved Frege to his investigations. According to Frege, if in our proofs of mathematical truths we only meet the laws of logic and definitions we have an "analytic" truth, but if it is not possible to carry out the proof

[1] Translated, with the exception of the Introductory Note, from Professor Frege's *Grundgesetze der Arithmetik* by Johann Stachelroth and Philip E. B. Jourdain.

[2] It may be mentioned here that the Open Court Publishing Company of Chicago and London has issued translations of the most important work of Dedekind (*Essays on the Theory of Numbers*) and Cantor (*Contributions to the Founding of the Theory of Transfinite Numbers*) in this direction.

without using principles which are not, in general, of a logical nature, but refer to a special domain of knowledge, the theorem is "synthetic." This distinction of Frege's is not quite Kant's distinction, but it is an extension of Kant's more limited view that the sole source of analytic judgments is the principle of contradiction. "If," said Frege, "we call a theorem *a posteriori* or analytic, we do not judge about the psychological, physiological and physical conditions which made it possible to form the content of the theorem in our consciousness, nor about how another person has—perhaps in an erroneous manner—arrived at maintaining its truth, but about the ultimate foundation of the justification for the maintenance of its truth.... A truth is *a posteriori* if its proof must depend on facts, that is to say, unprovable truths without generality that contain statements about definite objects. If, on the other hand, it is possible to carry out the proof wholly from general laws which themselves neither are capable of proof nor need it, the truth is *a priori*."[3] Of the four combinations, then, between analytic and synthetic on the one hand and *a priori* and *a posteriori* on the other, one only—analytic *a posteriori*—drops out.[4]

A still earlier account, written by Frege, is that he proposed to himself the question as to whether arithmetical judgments can be proved in a purely logical manner or must rest ultimately on facts of experience. Consequently he began by finding how far it was possible to go in arithmetic by inferences which depend merely on the laws of general logic. In order that nothing that is due to intuition should come in without being noticed, it was most important to preserve the unbrokenness of the chain of inferences; and ordinary language was found to be unequal to the accuracy required for this purpose.

Hence arose what Frege called his *Begriffsschrift*— a word that may be translated as "ideography"—which was described and shown in use in a small book published at Halle in 1879 under the title: *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens.* The fundamental idea of this book was the transference of the distinction of "variable" and "constant" from mathematical analysis to the wider domain of pure thought in general. In mathematics the distinction is not thoroughly carried out; but Frege's distinction was quite thorough.

[3] *Grundlagen der Arithmetik*, Breslau, 1884, pp. 3-4.
[4] *Ibid.*, p. 17.

He divided all the signs that he used into: (1) letters, "by which we can represent to ourselves different things," like those in the generally valid theorem in mathematics $(a+b)c=ac+bc$ and which serve principally to express *generality*; (2) signs which have quite a definite meaning, like +, —, 0, 1, or 2.

We have seen that arithmetic was the starting-point on the road that led Frege to his ideography. The aim of this ideography was not to provide a means of dealing systematically and rapidly with complicated logical questions, but to enable the question as to the empirical or purely logical basis of a branch of knowledge—in this case arithmetic—to be finally settled.

Frege began his *Begriffsschrift* by pointing out that, when we raise the question as to the foundation of a truth, the answer which —unlike that given by the recounting of the historical genesis and development of our knowledge of the truth in question—is connected with its inner being, consists in carrying out its proof purely logically, if that is possible, or, if it is not, in reducing it to the facts of experience on which the proof rests. The firmest proof is obviously a purely logical one, which, abstracting as it does from the special nature of things, is founded wholly on the laws on which all knowledge rests. Certainly a proposition may be capable of logical proof and yet could never, without sense-perception, enter into our consciousness. Indeed, this seems to be the case with *every* judgment, since no mental development without sense-perception appears possible. Thus it is not the psychological origin, but the completest manner of proof, that brings about the division of the class of all truths which need founding into (*a*) those which can be proved purely logically, and (*b*) those whose proofs rest on facts of experience.

And the very important ends for which Frege's ideography was designed were more or less overlooked by Venn, Schröder, and Peano, who criticized principally the cumbrousness of Frege's notation. This cumbrousness is a fact, but it may, as Bertrand Russell has shown, be avoided to a great extent. Far more important than the awkwardness of the form of many of the symbols, however, is the subtle and profound analysis of the ideas of logic, and the perfect avoidance of ambiguity and implicit assumptions. These are the most prominent characteristics of Frege's work.

In the following translation of part of Frege's mature exposition of 1893 and 1903, any notes or references which have been

added by myself are put in square brackets. It is to be hoped that the present translation, for which Professor Frege has most kindly given me his permission, will help to make Frege's magnificent work better known. Frege's work is the first of that of the modern logicians. Mr. Bertrand Russell in his "Lowell Lectures" of 1914,[5] has given a notable example of the "logical-analytic" method in philosophy of which "the first complete example is to be found in the writings of Frege," and this method is now becoming almost as widely known as its importance deserves.—P. E. B. J.]

THE ideal of a strictly scientific method in mathematics, which I have tried to realize here and which perhaps might be named after Euclid, I would like to describe in the following way.

It cannot be expected that we should prove everything, because that is impossible; but we can demand that all propositions used without proof should be expressly mentioned as such, so that we can see distinctly upon what the whole construction is founded. We should, then, strive to diminish the number of these fundamental laws as much as possible by proving everything that can be proved. Furthermore I demand—and that is where I go beyond Euclid—that all the methods of inference used must be specified in advance. Otherwise it is impossible to satisfy the first demand.

At this ideal I believe I have arrived in essentials: only in a few points could one possibly be more exacting. In order to assure myself of more freedom and in order not to drop into excessive prolixity, I have taken the liberty of making tacit use of the interchangeability of the minor terms (conditions) and of the possibility of amalgamation of identical minor terms, and I have not reduced the modes of inference to their smallest number. Those who have read my *Begriffsschrift* will be able to gather from it that it

[5] *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*, Chicago and London, 1914; cf. p. v.

would even in this respect be possible to satisfy the severest demand, but that it would at the same time involve a considerable increase in volume.

I believe that, apart from this, the only objections which could justly be raised to this book do not concern the rigor but only the choice of the course of the proofs and of the intermediate steps of the proofs. Often there are several modes of proof possible; I have not tried to adopt them all, and thus it is possible—even probable—that I have not always chosen the shortest. But let whoever has any fault to find with regard to this do better himself. There are other matters about which it is possible to dispute. Some might have preferred to increase the number of the modes of inference admitted and thereby to arrive at a greater mobility and brevity. But we have to stop somewhere if my ideal is approved of, and wherever we stop, people may say: "It would have been better to admit still more modes of inference."

By the uninterrupted connection of the chains of inference, each axiom, assumption, hypothesis, or whatever we like to call it, upon which a proof is founded, is brought to light, and so we gain a basis for judgment on the epistemological nature of the law proved. It has often been said that arithmetic is only a more highly developed logic; but that remains disputable as long as the proofs contain transitions from one proposition to another which are not performed according to acknowledged logical laws, but seem to be founded on intuitive kowledge. Only when these transitions are resolved into simple logical steps can we be sure that arithmetic is founded solely upon logic. I have gathered together everything that can facilitate the judgment as to whether the chains of inference are convincing and the buttresses firm. If any one perchance finds anything faulty, he must be able to indicate exactly where, to his thinking, the error lies—whether in the fun-

damental laws, in the definitions, in the rules, or in their application at a definite place. If we find everything correct, we know thus the exact bases upon which each single theorem is founded. A dispute can only, as far as I can see, arise because of my fundamental law of "ranges" (*Werthverläufe*),[6] which perhaps has not yet been specifically expressed by logicians, though it is in their minds when, for example, they speak of extensions (*Umfänge*) of concepts. I hold that it is purely logical. In any case the place is indicated where the decision has to be made.

My purpose requires many deviations from what is usual in mathematics. The requirements with regard to the rigor of proofs inevitably entail a great length of these proofs. Whoever does not think of this will often be surprised at the roundabout way in which a proposition is here proved, whereas he believes he can grasp the proof directly by a single act of understanding. This will surprise us especially if we compare the work of Dedekind, *Was sind und was sollen die Zahlen?*,[7] which is the most thorough work on the foundation of arithmetic that I have lately seen. In a much smaller compass it follows the laws of arithmetic much farther than I do here. This brevity is only arrived at, to be sure, by much not being really proved at all. Dedekind often says only that the proof follows from such-and-such theorems; he uses little dots which have the vague meaning of "and so on";[8] nowhere is there a statement of the logical or other laws on which he builds, and, even if there were, we could not possibly find out whether really no others were used,—for

[6] [This theorem is numbered V on pp. 36 and 240 of Vol. I (1893) of the *Grundgesetze*; and expresses that an equality of ranges both implies and is implied by the statement; thus an equation between functions holds quite generally. It first appeared on page 10 of Frege's lecture, *Funktion und Begriff* (Jena, 1891). Cf. p. 253 of Vol. II of the *Grundgesetze* (1903).]

[7] [English translation on pp. 29-115 of Dedekind's *Essays on the Theory of Numbers* (Chicago and London, 1901).]

[8] [Cf., for example, paragraph 8 on page 47 of the above translation.]

to do that the proof must be not merely indicated but completely carried out. Dedekind is also of the opinion that the theory of number is a part of logic; but his work hardly contributes to strengthen this opinion, because the expressions "system" and "a thing belongs to a thing" used by him are not usual in logic and are not reducible to accepted logical doctrine. I do not say this as a reproach, for his method may have been the most serviceable to him for his purpose; I only say it to make my intentions clear by putting them by the side of opposite intentions. The length of a proof is not to be measured by the yard. It is easy to make a proof appear short on paper by omitting many connecting links in the chain of inference and only indicating many things. Generally we are satisfied if every step in the proof is seen to be correct, and we may be so if we intend to arouse conviction of the truth of the theorem to be proved. If we wish to bring about an insight into the nature of this perception of the truth this method does not suffice, but we must put down all the intermediate stages of reasoning, in order that the full light of consciousness may fall upon them. As a rule mathematicians are only interested in the content of a theorem and in the fact that it is to be proved. The novelty of this book does not lie in the content of the theorems but in the development of the proofs and the foundations upon which they are based. That this altogether different point of view needs a quite different treatment ought not to appear strange. If we deduce one of our theorems in the usual way, it will be easy to overlook a proposition which does not appear necessary for the proof. If my proof is carefully thought out, the indispensability of this proposition will, I believe, be seen, unless an altogether different mode of procedure is adopted. Thus perhaps there are here and there in our theorems conditions which appear at first to be unnecessary but which after all prove to be necessary or at least to

admit of removal only by a proposition to be specially proved.

With this book I accomplish an object which I had in view in my *Begriffsschrift* of 1879 and which I announced in my *Grundlagen der Arithmetik*.[9] I will here substantiate the opinion on the concept of number that I expressed in the book last mentioned. The fundamental part of my results is there expressed in § 46 in the words that the numerical datum contains an assertion about a concept; and upon this my present work is founded. If anybody is of another opinion let him try to construct a logical and usable exposition of his view by signs, and he will see that it is impossible. In language, it is true, the state of affairs is not so obvious, but if we look into the matter closely we find that here too a numerical datum always denotes a concept, not a group, an aggregate[10] or such-like things; and that if a group or aggregate is named, it is always determined by a concept, that is to say, by the properties an object must have in order to belong to the group, while that which makes the group a group or the system a system —the relations of members to each other—is altogether indifferent for the number.

The reason why the demonstration appears so long after the enunciation is to be found in part in essential changes of my ideography, which have forced me to discard a manuscript that was almost completed. These improvements may be mentioned here briefly. The fundamental signs employed in my *Begriffsschrift* have with one exception been used again here. Instead of the three parallel lines I have chosen the ordinary sign of equality because I convinced myself that it has exactly the same meaning in arithmetic that I wish to designate. I use the ex-

[9] Cf. the introduction and §§ 90 and 91 of my *Grundlagen der Arithmetik*, Breslau, 1884.

[10] [However, "aggregate" has become a technical term in mathematics for the translation of *"Menge"* or *"Begriffsumfang,"* and not *"Aggregat."*]

pression "equal" in the same sense as "coinciding with" or "identical with," and this is just how the sign of equality is really used in arithmetic. The objection which might perhaps be raised against this rests on a defective distinction between sign and what is signified. It is true that in the equation $2^2 = 2 + 2$ the sign on the left is different from the one on the right, but both indicate or denote the same number.[11] To the old fundamental signs two more have been added: the "smooth breathing" (*spiritus lenis*) which serves for the designation of the "range" (*Werthverlauf*) of a function, and a sign which is meant to take the place of the definite article of ordinary language. The introduction of the ranges of functions is an important advance which makes possible a far greater flexibility. The former derived signs can now be replaced by other and simpler ones, though the definitions of one-to-one-ness, of a relation, of succession in a series, and of representation (*Abbildung*) are essentially the same as those which I have given partly in my *Begriffsschrift* and partly in my *Grundlagen der Arithmetik*. But the ranges have also a great fundamental importance; in fact I even define number itself as the extension of a concept, and extensions of concepts are, according to my definition, ranges. In consequence, we cannot do without them. The old fundamental signs, which reappear outwardly unchanged and whose algorithm has also hardly changed, have nevertheless been supplied with other explanations. The former "line of content" (*Inhaltsstrich*) reappears as a horizontal line (*Wagerechter*). There are consequences of an energetic development of my logical views. Formerly I distinguished in that proposition whose outer form is an assertion two things: (*a*) The recognition of the truth; (*b*) The content which is recognized as true. The content I called the "judicable content" (*beurtheil-*

[11] I also say: the meaning (*Sinn*) of the sign on the right is different from that of the one on the left but the denotation (*Bedeutung*) is the same (*Zeitschr. für Philos. und philos. Kritik*, Vol. C, 1892, pp. 25-50).

barer Inhalt). The latter has been divided into what I call "thought" (*Gedanken*) and "truth-value" (*Wahrheitswerth*). That is a consequence of the distinction between the meaning and denotation of a sign. In this case the meaning of a proposition is the thought and its denotation the truth-value. Besides this, we must grant that the truth-value is the true. I distinguish two truth-values: the true and the false. This distinction I have discussed more exhaustively and substantiated in my above mentioned essay on meaning and denotation. It may be mentioned here that incorrect speech can only thus be rightly understood. The thought which is otherwise the meaning of a proposition becomes, in incorrect speech, its denotation. How much more simple and distinct everything becomes by the introduction of truth-values can only be seen by an exhaustive examination of this book. These advantages alone put a great weight into the balance in favor of my view, which view perhaps may seem strange at first sight. Also the essence of the *function* in contradistinction from the *object* (*Gegenstand*) is more distinctly accentuated than in my *Begriffsschrift*. From this results further the distinction of function of the first and second "stage" (*Stufe*). As I have shown in my essay *Funktion und Begriff*, published at Jena in 1891, relations are functions in the meaning of the word which has been extended by me, and so we have to distinguish concepts of the first and second stage, relations of the same and of different stages.

From this it will be seen that the years have not passed in vain since the appearance of my *Begriffsschrift* and *Grundlagen*: they have brought my work to maturity. But just that which I recognize as an important advance stands, as I cannot help seeing, as a great obstacle in the way of the circulation and effectiveness of my book. And the strict completeness of the chain of conclusions, which seems to my way of thinking not its least value, will bring

it, I am afraid, little thanks. I have got farther away from the traditional ideas and have by so doing given an appearance of paradox to my views. An expression which is encountered here and there on rapidly turning over these pages may easily appear strange and produce an unfavorable impression. I myself can judge somewhat with what opposition my innovations will be met because I have had to overcome something similar in myself. For not at random or because of the desire for innovation did I arrive at them, but I was forced by the matter itself.

With this I arrive at the second reason for my delay: the discouragement which at times came over me because of the cool reception, or rather the want of reception, by mathematicians,[12] of my works mentioned above and the opposing scientific currents against which my book would have to fight. Even the first impression must frighten people away: unknown signs, pages of nothing but strange-looking formulas. It is for that reason that I turned at times toward other subjects. But I could not keep the results of my thinking which seemed valuable to me myself locked up in my desk for any length of time; and the labor I had spent always required renewed labor that it might not be in vain. So the subject did not let go its hold upon me. In a case like the present one, when the value of a book cannot be recognized by a hasty perusal, criticism ought to be a help. But criticism is generally too badly paid. A critic can never hope to get paid in cash for the pains which the thorough study of this book will cost him. The only remaining hope is that somebody may have beforehand sufficient confidence in the matter to expect that the subjective gain will be sufficient recompense, and that

[12] In vain do we seek a notice of my *Grundlagen der Arithmetik* in the *Jahrbuch über die Fortschritte der Mathematik*. Investigators in the same domain, Dedekind, Otto Stolz, and von Helmholtz, do not seem to know my works. Nor does Kronecker mention them in his essay on the concept of number.

he will then publish the results of his searching examination. It is not as if only a laudatory review would satisfy me: quite the contrary. I would by far prefer an attack supported by a thorough acquaintance with the subject than to be praised in general terms which do not touch the root of the matter....

I must give up hope of securing as readers all those mathematicians who, when they come across logical expressions like "concept," "relation," "judgment," think: *Metaphysica sunt, non leguntur*; and those philosophers who at the sight of a formula call out: *Mathematica sunt, non leguntur.* Perhaps the number of these people is not very small. Perhaps also the number of mathematicians who trouble themselves about the foundation of their science is not great, and even those who do often seem in a great hurry to get past the foundations. And I hardly dare hope that my reasons for laborious rigor and consequent lengthiness will convince many of them. As we know, what is long established has great power over the minds of men. If I compare arithmetic with a tree which develops at the top into a multitude of methods and theorems while the root pushes downward, it seems to me that the pushing of the root is, at least in Germany, rather weak. Even in a work which might be classed among those dealing with foundations, the *Algebra der Logik* of E. Schröder, the top-growth soon predominates and, even before a great depth has been reached, causes a bending upward and a development into methods and theorems.

The widespread inclination to recognize only what can be perceived by the senses as existing is also unfavorable for my book. It is sought to deny, or at least to overlook, what cannot be thus perceived. Now the objects of arithmetic, that is to say numbers, are of a kind which cannot be thus perceived. How are we to deal with them? Very simply: the signs used for the numbers are explained to

be the numbers themselves. Then in the signs there is something visible, and that is the chief thing. No doubt the signs have altogether different properties from the numbers themselves, but what does that matter? We simply ascribe to them the desired properties by means of what we call definitions. How on earth there can be a definition where there is no question about connections between sign and what is signified by it is a puzzle. We knead together sign and what is signified as far as possible without making any distinction between them, and, according to circumstances, we can assert the existence of the result with mention of its tangibility,[13] or we can bring into prominence the actual properties of numbers. Sometimes these number-signs are, it seems, regarded as chessmen and the so-called definitions as rules of the game. The sign then does not signify anything, but is the subject-matter itself. It is true that in this we overlook one little thing: that is, that we express a thought by $3^2 + 4^2 = 5^2$, while a position of chessmen does not express anything. Where people are satisfied with such superficialities, there is of course no basis for a deeper understanding.

Here it is of importance to make clear what defining is and what we can reach by it. It is, it seems, often credited with a creative power while really all there is to defining is that something is brought out, precisely limited and given a name. The geographer does not create a sea when he draws border lines and says: The part of the surface of the water surrounded by these lines, I am going to call the Yellow Sea; and no more can the mathematician really create anything by this process of definition. Nor can we by a mere definition magically give to a thing a property which it has not got. All we can do is to call

[13] Cf. E. Heine, "Die Elemente der Funktionslehre," Crelle's *Journal für Math.*, Vol. LXXIV, p. 173: "I place myself in my definition in a purely formal position and call certain tangible signs numbers so that in consequence the existence of these numbers is not in question.

this particular property by a new name. But that an oval drawn on paper with pen and ink should acquire by definition the property that when it is added to one one results, I can only regard as a scientific superstition. One might just as well make a lazy pupil diligent by a mere definition. Confusion easily arises here through lack of a distinction between concept (*Begriff*) and object (*Gegenstand*). If we say: "A square is a rectangle in which the adjacent sides are equal," we define the concept *square* by indicating what properties something must have in order to fall under this concept. I call these properties "characteristics" (*Merkmale*) of the concept. But it must be carefully noted that these characteristics are not the properties of the concept. The concept *square* is not a rectangle, only the objects which fall under this concept are rectangles, just as the concept *black cloth* is neither black nor a cloth. Whether or not such objects exist is not immediately known by means of their definitions. Now, for instance, suppose that we wish to define the number zero by saying: "It is something which when added to one gives one." With that we have defined a concept by stating what property an object must have to fall under this concept. But this property is not a property of the concept defined. It seems that we often imagine that we have created by our definition something which when added to one gives one. This is a delusion. Neither has the concept defined this property, nor is the definition a guarantee that the concept is satisfied. That requires first of all an investigation. Only when we have proved that there exists one object and one only with the required property are we in a position to give this object the proper name "zero." To create the zero is consequently impossible. I have already repeatedly explained this but, as it seems, without result.

JENA, GERMANY. GOTTLOB FREGE.

OUGHTRED'S IDEAS AND INFLUENCE ON THE TEACHING OF MATHEMATICS.*

GENERAL STATEMENT.

WILLIAM OUGHTRED has nowhere given a full and systematic exposition of his views on mathematical teaching. Nevertheless, he had very pronounced and clear cut ideas on the subject. That a man who was not a teacher by profession should have mature views on teaching is most interesting. We gather his ideas from the quality of the books he published, from his prefaces and from passages in his controversial writing against Delamain. As we proceed to give quotations unfolding Oughtred's views, we shall observe that three points receive special emphasis:

1. An appeal to the eye through suitable symbolism;
2. Emphasis upon rigorous thinking;
3. The postponement of the use of mathematical instruments until after the logical foundations of a subject have been thoroughly mastered.

The importance of these tenets is immensely reinforced by the conditions of the hour. This voice from the past speaks wisdom to specialists of to-day. Recent methods of determining educational values and the modern cult of utilitarianism have led some experts to extraordinary conclusions. Laboratory methods of testing, by the narrow-

*For details of William Oughtred's life we refer to *The Open Court* of August, 1915, and for a description of his works to *The Monist* of July, 1915.

ness of their range, often mislead. Thus far they have been inferior to the word of a man of experience, insight and conviction.

MATHEMATICS, "A SCIENCE OF THE EYE."

Oughtred was a great admirer of the Greek mathematicians—Euclid, Archimedes, Apollonius of Perga, Diophantus. But in reading their words he experienced keenly what many modern readers have felt, namely, that the almost total absence of mathematical symbols renders their writings unnecessarily difficult to read. Statements that can be compressed into a few well-chosen symbols which the eye is able to survey as a whole are expressed in long drawn out sentences. A striking illustration of the importance of symbolism is afforded by the history of the formula

$$ix = \log(\cos x + i \sin x).$$

It was given in Roger Cotes's *Harmonia mensurarum*, 1722, not in symbols, but expressed in rhetorical form, destitute of special aids to the eye. The result was that the theorem remained in the book undetected for 185 years and was meanwhile re-discovered by others. Owing to the prominence of Cotes as a mathematician it is very improbable that such a thing could have happened, had the theorem been thrust into view by the aid of mathematical symbols.

In studying the ancient authors Oughtred is reported to have written down on the margin of the printed page some of the theorems and their proofs, expressed in the symbolic language of algebra.

In the preface of his *Clavis* of 1631 and of 1647 he says:

"Wherefore, that I might more clearly behold the things themselves, I uncasing the Propositions and Demonstrations out of their covert of words, designed them in

notes and species appearing to the very eye. After that by comparing the divers affections of Theorems, inequality, proportion, affinity, and dependence, I tryed to educe new out of them."

It was this motive which led him to introduce many abbreviations in algebra and trigonometry. The pedagogical experience of recent centuries has endorsed Oughtred's view, provided of course that the pupil is carefully taught the exact meaning of the symbols. There have been and there still are those who oppose the intensive use of symbolism. In our day the new symbolism for all mathematics, suggested by the school of Peano in Italy, can hardly be said to be received with enthusiasm. In Oughtred's day symbolism was not yet the fashion. To be convinced of this fact one need only open a book of Edmund Gunter, with whom Oughtred came in contact in his youth, or consult the *Principia* of Sir Isaac Newton who flourished after Oughtred. The mathematical works of Gunter and Newton, particularly the former, are surprisingly destitute of mathematical symbols. The philosopher Hobbes, in a controversy with John Wallis, criticized the latter for that "Scab of Symbols," whereupon Wallis replied, "I wonder how you durst touch M. Oughtred for fear of catching the Scab. For, doubtlesse, his book is as much covered over with the Scab of Symbols, as any of mine.... As for my Treatise of Conick Sections, you say, it is covered over with the Scab of Symbols, that you had not the patience to examine whether it is well or ill demonstrated."[1]

Oughtred maintained his view of the importance of symbols on many different occasions. Thus, in his *Circles of Proportion*, 1632, p. 20:

"This manner of setting downe Theoremes, whether

[1] *Due Correction for Mr. Hobbes. Or Schoole Discipline, for not saying his Lessons right. In answer to his Six Lessons, directed to the Professors of Mathematicks.* By the Professor of Geometry. Oxford, 1656, pp. 7, 47, 50.

they be Proportions, or Equations, by Symbols or notes of words, is most excellent, artificiall, and doctrinall. Wherefore I earnestly exhort every one, that desireth though but to looke into the noble Sciences Mathematicall, to accustome themselves unto it: and indeede it is easie, being most agreeable to reason, yea even to sense. And out of this working may many singular consectaries be drawne: which without this would, it may be, for ever lye hid."

RIGOROUS THINKING AND THE USE OF INSTRUMENTS.

The author's elevated concept of mathematical study as conducive to rigorous thinking shines through the following extract from his preface to the 1647 *Clavis*:

"...Which Treatise being not written in the usuall synthetical manner, nor with verbous expressions, but in the inventive way of Analitice, and with symboles or notes of things instead of words, seemed unto many very hard; though indeed it was but their owne diffidence, being scared by the newnesse of the delivery; and not any difficulty in the thing itselfe. For this specious and symbolicall manner, neither racketh the memory with multiplicity of words, nor chargeth the phantasie with comparing and laying things together; but plainly presenteth to the eye the whole course and processe of every operation and argumentation.

"Now my scope and intent in the first Edition of that my Key was, and in this New Filing, or rather forging of it, is, to reach out to the ingenious lovers of these Sciences, as it were Ariadnes thread, to guide them through the intricate Labyrinth of these studies, and to direct them for the more easie and full understanding of the best and antientest Authors;.... That they may not only learn their propositions, which is the highest point of Art that most Students aime at; but also may perceive with what solertiousnesse, by what engines of aequations, Interpre-

tations, Comparations, Reductions, and Disquisitions, those antient Worthies have beautified, enlarged, and first found out this most excellent Science.... Lastly, by framing like questions problematically, and in a way of Analysis, as if they were already done, resolving them into their principles, I sought out reasons and means whereby they might be effected. And by this course of practice, not without long time, and much industry, I found out this way for the helpe and facilitation of Art."

Still greater emphasis upon rigorous thinking in mathematics is laid in the preface to the *Circles of Proportion* and in some parts of his *Apologeticall Epistle* against Delamain. In that preface William Forster quotes the reply of Oughtred to the question how he (Oughtred) had for so many years concealed his invention of the slide rules from himself (Forster) whom he had taught so many other things. The reply was:

"That the true way of Art is not by Instruments, but by Demonstration: and that it is a preposterous course of vulgar Teachers, to begin with Instruments, and not with the Sciences, and so instead of Artists, to make their Scholers only doers of tricks, and as it were Iuglers: to the despite of Art, losse of previous time, and betraying of willing and industrious wits, unto ignorance, and idlenesse. That the vse of Instruments is indeed excellent, if a man be an Artist: but contemptible, being set and opposed to Art. And lastly, that he meant to commend to me, the skill of Instruments, but first he would have me well instructed in the Sciences."

Delamain took a different view, arguing that instruments might very well be placed in the hands of pupils from the start. At the time of this controversy Delamain supported himself by teaching mathematics in London and he advertised his ability to give instruction in mathematics, including the use of instruments. Delamain brought the

charge against Oughtred of unjustly calling "many of the [British] Nobility and Gentry doers of trickes and jugglers." To this Oughtred replies:[2]

"As I did to Delamain and to some others, so I did to William Forster: I freely gave him my helpe and instruction in these faculties: only this was the difference, I had the very first moulding (as I may say) of this latter: But Delamain was already corrupted with doing upon Instruments, and quite lost from ever being made an Artist: I suffered not William Forster for some time so much as speake of any Instrument, except only the Globe it selfe; and to explicate, and worke the questions of the Sphaere, by the way of the Analemma: which also himselfe did describe for the present occasion. And this my restraint from such pleasing avocations, and holding him to the strictnesse of percept, brought forth this fruit, that in short time, even by his owne skill, he could not onely use any Instrument he should see, but also was able to delineate the like, and devise others."

As representing Delamain's views, we make the following selection from his *Grammelogia* (London, about 1633), the part near the end of the book and bearing the title, "In the behalfe of vulgar Teachers and others," where Delamain refers to Oughtred's charge that the scholars of "vulgar" teachers are "doers of tricks, as it were iuglers." Delamain says:

". . . . Which words are neither *cautelous,* nor *subterfugious,* but are as downe right in their *plainnesse,* as they are touching, and *pernitious,* by two much derogating from many, and glancing upon many noble *personages,* with too *grosse,* if not too *base* an attribute, in tearming them *doers of tricks, as it were to iuggle*: because they perhaps make use of a necessitie in the furnishing of themselves with such knowledge by *Practicall Instrumentall operation,*

[2] Oughtred, *Apologeticall Epistle,* p. 27.

when their more weighty *negotiations* will not permit them for *Theoreticall figurative demonstration*; those that are guilty of the aspertion, and are touched therewith may answer for themselves, and studie to be more *Theoreticall,* than *Practicall*: for the *Theory,* is as the *Mother* that produceth the *daughter,* the very sinewes and life of *Practise,* the excellencie and highest degree of true *Mathematicall Knowledge*: but for those that would make but a step as it were into that kind of *Learning,* whose onely desire is expedition, and facilitie, both which by the generall consent of all are best effected with Instrument rather than with tedious regular demonstrations, it was ill to checke them so grosly, not onely in what they have *Practised,* but abridging them also of their liberties with what they may *Practise,* which aspertion may not easily be slighted off by any *glosse* or *Apologie,* without an Ingenuous *confession,* or some mentall reservation: To which vilification, howsoever, in the behalfe of my selfe, and others, I answer; That *Instrumentall* operation is not only the Compendiating, and facilitating of *Art,* but even the glory of it, whole demonstration both of the making, and operation is soly in the *science,* and to an *Artist* or disputant proper to be knowne and so to all, who would truly know the cause of the *Mathematicall operations* in their originall; But, for none to know the use of a *Mathematicall Instrumen*[*t*], except he knowes the cause of its operation, is somewhat too strict, which would keepe many from affecting the *Art,* which of themselves are ready enough every where, to conceive more harshly of the difficultie, and impossibilitie of attayning any skill therein, than it deserves, because they see nothing but obscure propositions, and perplex and intricate demonstrations before their eyes, whose unsavoury tartnes, to an unexperienced palate like bitter pills is sweetned over, and made pleasant with an *Instrumentall compendious facilitie,* and made to goe downe the more readily, and

yet to retaine the same vertue, and working; And me thinkes in this queasy age, all *helpes* may bee used to procure a *stomacke,* all *bates* and invitations to the declining studie of so noble a *Science,* rather than by rigid Method and generall *Lawes* to scarre men away. All are not of like disposition, neither all (as was sayd before) propose the same end, some resolve to *wade,* others to put a *finger* in onely, or wet a *hand*: now thus to tye them to an obscure and *Theoricall* forme of teaching, is to crop their hope, even in the very bud. . . . The beginning of a *man's knowledge* even in the use of an *Instrument,* is first founded on *doctrinal precepts,* and these precepts may be conceived all along in its use: and are so farre from being excluded, that they doe necessarily *concomitate* and are contained therein: the *practicke* being better understood by the *doctrinall part,* and this later explained by the *Instrumentall,* making precepts obvious unto sense, and the *Theory* going along with the *Instrument,* better informing and inlightning the understanding, etc. *vis vnita fortior,* so as if that in *Phylosophy* bee true, *Nihil est [in] intellectu quod non prius fuit in sensu.*"

The difference between Oughtred and Delamain as to the use of mathematical instruments raises an important question. Should the slide rule be placed in the hands of a boy before, or after, he has mastered the theory of logarithms? Should logarithmic tables be withheld from him until the theoretical foundation is laid in the mind of the pupil? Is it a good thing to let a boy use a surveying instrument unless he first learns trigonometry? Is it advisable to permit a boy to familiarize himself with the running of a dynamo before he has mastered the underlying principles of electricity? These and similar questions are even more vital to-day than they were in the seventeenth century. Does the use of instruments ordinarily discourage a boy from mastery of the theory? Or does such manipulation con-

stitute a natural and pleasing approach to the abstract? On this particular point, who showed the profounder psychological insight, Oughtred or Delamain?

In July, 1914, there was held in Edinburgh a celebration of the three hundredth anniversary of the invention of logarithms. On that occasion there was collected at Edinburgh university one of the largest exhibits ever seen of modern instruments of calculation. The opinion was expressed by an experienced teacher that "weapons as those exhibited there are for men and not for boys, and such danger as there may be in them is of the same character as any form of too early specialization."

It is somewhat of a paradox that Oughtred who in his student days and during his active years felt himself impelled to invent sun-dials, planispheres and various types of slide rules—instruments which represent the most original contributions which he handed down to posterity—should discourage the use of such instruments in teaching mathematics to beginners. That without the aid of instruments he himself should have succeeded so well in attracting and inspiring young men constitutes the strongest evidence of his transcendent teaching ability. It may be argued that his pedagogic dogma, otherwise so excellent, here goes contrary to the course he himself followed instinctively in his self-education along mathematical lines. We read that Sir Isaac Newton, as a child, constructed sun-dials, wind-mills, kites, paper lanterns and a wooden clock. Should these activities have been suppressed? Ordinary children are simply Isaac Newtons on a smaller intellectual scale. Should their activities along these lines be encouraged or checked?

On the other hand it may be argued that the paradox alluded to above admits of explanation like all paradoxes, and that there is no inconsistency between Oughtred's pedagogic views and his own course of development. If he

invented sun-dials, he must have had a comprehension of the cosmic motions involved; if he solved spherical triangles graphically by the aid of the planisphere, he must have understood the geometry of the sphere, so far as it relates to such triangles; if he invented slide rules, he had beforehand a thorough grasp of logarithms. The question at issue does not involve so much the invention of instruments, as the use by the pupil of instruments already constructed, before he fully understands the theory which is involved. Nor does Sir Isaac Newton's activity as a child establish Delamain's contention. Of course, a child should not be discouraged from manual activity along the line of producing interesting toys in imitation of structures and machines that he sees, but to introduce him to the realm of abstract thought by the aid of instruments is a different proposition, fraught with danger. A boy may learn to use a slide rule mechanically and, because of his ability to obtain practical results, feel justified in foregoing the mastery of underlying theory; or he may consider the ability of manipulating a surveying instrument quite sufficient, even though he be ignorant of geometry and trigonometry; or he may learn how to operate a dynamo and an electric switchboard, and be altogether satisfied, though having no grasp of electrical science. Thus instruments draw a youth aside from the path leading to real intellectual attainments and real efficiency; they allure him into lanes which are often blind alleys. Such were the views of Oughtred.

Who was right, Oughtred or Delamain? It may be claimed that there is a middle ground which more nearly represents the ideal procedure in teaching. Shall the slide rule be placed into the student's hands at the time when he is engaged in the mastery of principles? Shall there be an alternate study of the theory of logarithms and of the slide rule — on the idea of one hand washing the other—until a mastery of both the theory and the use of the

instrument has been attained? Does this method not produce the best and most lasting results? Is not this Delamain's actual contention? We leave it to the reader to settle these matters from his own observation, knowledge and experience.

NEWTON'S COMMENTS ON OUGHTRED.

Oughtred is an author who has been found to be of increasing interest to modern historians of mathematics. But no modern writer has, to our knowledge, pointed out his importance in the history of the *teaching* of mathematics. Yet his importance as a teacher did receive recognition in the seventeenth century by no less distinguished a scientist than Sir Isaac Newton. On May 25, 1694, Sir Isaac Newton wrote a long letter in reply to a request for his recommendation on a proposed new course of study in mathematics at Christ's Hospital.[3] Toward the close of his letter, Newton says:

"And now I have told you my opinion in these things, I will give you Mr. Oughtred's, a Man whose judgment (if any man's) may be safely relyed upon. For he in his book of the circles of proposition, in the end of what he writes about Navigation (page 184) has this exhortation to Seamen. And if, sayth he, the Masters of Ships and Pilots will take the pains in the Journals of their Voyages diligently and faithfully to set down in severall columns, not onely the Rumb they goe on and the measure of the Ships way in degrees, and the observation of Latitude and variation of their compass; but alsoe their conjectures and reason of their correction they make of the aberrations they shall find, and the qualities and condition of their ship, and the diversities and seasons of the winds, and the secret motions or agitations of the Seas, when they begin, and how long they continue, how farr they extend and with

[3] J. Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes*, London, 1850, pp. 279-292.

what inequality; and what else they shall observe at Sea worthy consideration, and will be pleased freely to communicate the same with Artists such as are indeed skilfull in the Mathematicks and lovers and enquirers of the truth: I doubt not but that there shall be in convenient time, brought to light many necessary precepts which may tend to the perfecting of Navigation, and the help and safety of such whose Vocations doe inforce them to commit their lives and estates in the vast Ocean to the providence of God. Thus farr that very good and judicious man Mr. Oughtred, I will add, that if instead of sending the Observations of Seamen to able Mathematicians at Land, the Land would send able Mathematicians to Sea, it would signify much more to the improvement of Navigation and safety of Mens lives and estates on that element."

May Oughtred prove as instructive to the modern reader as he did to Newton.

OUGHTRED AND HARRIOT.

Oughtred's *Clavis mathematicae* was the most influential mathematical publication in Great Britain which appeared in the interval between John Napier's *Mirifici logarithmorum canonis descriptio,* Edinburgh, 1614, and the time, forty years later, when John Wallis began to publish his important researches at Oxford. The year 1631 is of interest as the date of publication, not only of Oughtred's *Clavis,* but also of Thomas Harriot's *Artis analyticae praxis.* We have no evidence that these two mathematicians ever met. Through their writings they did not influence each other. Harriot died ten years before the appearance of his *magnum opus,* or ten years before Oughtred began to publish. Strangely, Oughtred who survived Harriot thirty-nine years, never mentions him. There is no doubt that, of the two, Harriot was the more original mind, more capable of penetrating into new fields

of research. But he had the misfortune of having a strong competitor in René Descartes, in the development of Algebra, so that no single algebraic achievement stands out strongly and conspicuously as Harriot's own contribution to algebraic science. As a text to serve as an introduction to algebra, Harriot's *Artis analyticae praxis* was inferior to Oughtred's *Clavis*. The former was a much larger book, not as conveniently portable, compiled after the author's death by others and not prepared with the care in the development of the details, nor with the coherence and unity, and the profound pedagogic insight, which distinguish the work of Oughtred. Nor was Harriot's position in life such as to be surrounded by so wide a circle of pupils as was Oughtred. To be sure, Harriot had such followers as Torperley, William Lower and Protheroe in Wales, but this group is small as compared with Oughtred's.

OUGHTRED'S PUPILS.

There was a large number of distinguished men who, in their youth, either visited Oughtred's home and studied under his roof or else read his *Clavis* and sought his assistance by correspondence. We permit Aubrey to enumerate some of these pupils in his own gossipy style:[4]

"Seth Ward, M.A., a fellow of Sydney Colledge in Cambridge (now bishop of Sarum), came to him and lived with him halfe a yeare (and he would not take a farthing for his diet), and learned all his mathematiques of him. Sir Jonas More was with him a good while, and learn't; he was but an ordinary logist before. Sir Charles Scarborough was his scholar; so Dr. John Wallis was his scholar; so was Christopher Wren his scholar, so was Mr.... Smethwyck, Regiae Societatis Socius. One Mr. Austin (a most ingeniose man) was his scholar, and studyed so

[4] Aubrey, *op. cit.*, Vol. II, 1898, p. 108.

much that he became mad, fell a laughing, and so dyed, to the great griefe of the old gentleman. Mr.... Stokes, another scholar, fell mad, and dream't that the good old gentleman came to him, and gave him good advice, and so he recovered, and is still well. Mr. Thomas Henshawe, Regiae Societatis Socius, was his scholar (then a young gentleman). But he did not so much like any as those that tugged and tooke paines to worke out questions. He taught all free.

"He could not endure to see a scholar write an ill hand; he taught them all presently to mend their hands."

Had Oughtred been the means of guiding the mathematical studies of only John Wallis and Christopher Wren —one the greatest English mathematician between Napier and Newton, the other one of the greatest architects of England,—he would have earned profound gratitude. But the above list embraces nine men, most of them distinguished in their day. And yet Aubrey's list is very incomplete. It is easy to more than double it by adding the names of William Forster who translated from Latin into English Oughtred's *Circles of Proportion,* Arthur Haughton who brought out the 1660 Oxford edition of the *Circles of Proportion,* Robert Wood, an educator and politician who assisted Oughtred in the translation of the *Clavis* from Latin into English for the edition of 1647, W. Gascoigne, a man of promise who fell, 1644, at Marston Moor, John Twysden who was active as a publisher, William Sudell, N. Ewart, Richard Shuttleworth, William Robinson, and Henry Frederick Howard who was the son of the Earl of Arundel, for whose instruction Oughtred originally prepared the manuscript treatise that was published in 1631 as the *Clavis mathematicae.*

Nor must we overlook the names of Lawrence Rook (who "did admirably well read in Gresham Coll. on the sixth chapt. of the said book," the *Clavis,*), Christopher

Brooke (a maker of mathematical instruments who married a daughter of the famous mathematician), William Leech and William Brearly (who with Robert Wood "have been ready and helpfull incouragers of me [Oughtred] in this labour" of preparing the English *Clavis* of 1647) and Thomas Wharton who studied the *Clavis* and assisted in the editing of the *Clavis* of 1647.

The devotion of these pupils bears eloquent testimony not only of Oughtred's ability as a mathematician but also of his power of drawing young men to him—of his personal magnetism. Nor should we omit from the list Richard Delamain, a teacher of mathematics in London, who unfortunately had a bitter controversy with Oughtred on the priority and independence of the invention of the circular slide rule and a form of sun-dial. Delamain became later a tutor in mathematics to King Charles I, and perished in the civil war, before 1645.

OUGHTRED, THE "TODHUNTER OF THE SEVENTEENTH CENTURY."

To afford a clearer view of Oughtred as a teacher and mathematical expositor we quote some passages from various writers and from his correspondence. Anthony Wood[5] gives an interesting account of how Seth Ward and Charles Scarborough went from Cambridge University to the obscure home of the country mathematician, to be initiated into the mysteries of algebra:

"Mr. Cha. Scarborough, then an ingenious young student and fellow of Caius Coll. in the same university, was his [Seth Ward's] great acquaintance, and both being equally students in that faculty and desirous to perfect themselves, they took a journey to Mr. Will. Oughtred living then at Albury in Surrey, to be informed in many things in his *Clavis mathematica* which seemed at that

[5] Wood's *Athenae Oxonienses* (Ed. P. Bliss), Vol. IV, 1820, p. 247.

time very obscure to them. Mr. Oughtred treated them with great humanity, being very much pleased to see such ingenious young men apply themselves to these studies, and in short time he sent them away well satisfied in their desires. When they returned to Cambridge, they afterwards read the *Clav. Math.* to their pupils, which was the first time that that book was read in the said university. Mr. Laur. Rook, a disciple of Oughtred, I think, and Mr. Ward's friend, did admirably well in Gresham Coll. on the sixth chap. of the said book, which obtained him great repute from some and greater from Mr. Ward, who ever after had an especial favour for him."

Anthony Wood makes a similar statement about Thomas Henshaw:[6]

"While he remained in that coll. [University College, Oxford] which was five years....he made an excursion for about 9 months to the famous mathematician Will. Oughtred parson of Aldbury in Surrey, by whom he was initiated in the study of mathematics, and afterwards retiring to his coll. for a time, he at length went to London, was entered a student in the Middle Temple."

Extracts from letters of W. Gascoigne to Oughtred, of the years 1640 and 1641, throw some light upon mathematical teaching of the time:[7]

"Amongst the mathematical rarities these times have afforded, there are none of that small number I (a late intruder into these studies) have yet viewed, which so fully demonstrates their authors' great abilities as your Clavis, not richer in augmentations, than valuable for contraction;...."

"Your belief that there is in all inventions aliquid divinum, an infusion beyond human cogitations, I am confident will appear notably strengthened, if you please to afford this truth belief, that I entered upon these studies acciden-

[6] Wood, *op. cit.*, Vol. II, p. 445. [7] Rigaud, *op. cit.*, Vol. I, pp. 33, 35.

tally after I betook myself to the country, having never had so much aid as to be taught addition, nor the discourse of an artist (having left both Oxford and London before I knew what any proposition in geometry meant) to inform me what were the best authors."

The following extracts from two letters by W. Robinson, written before the appearance of the 1647 English edition of the *Clavis,* express the feeling of many readers of the *Clavis* on its extreme conciseness and brevity of explanation:[8]

"I shall long exceedingly till I see your *Clavis* turned into a pick-lock; and I beseech you enlarge it, and explain it what you can, for we shall not need to fear either tautology or superfluity; you are naturally concise, and your clear judgment makes you both methodical and pithy; and your analytical way is indeed the only way.". . . .

"I will once again earnestly entreat you, that you be rather diffuse in the setting forth of your English mathematical *Clavis,* than concise, considering that the wisest of men noted of old, and said stultorum infinitus est numerus, these arts cannot be made too easy, they are so abstruse of themselves, and men either so lazy or dull, that their fastidious wits take a loathing at the very entrance of these studies, unless it be sweetened on with plainness and facility. Brevity may well argue a learned author that without any excess or redundance, either of matter or words, can give the very substance and essence of the thing treated of; but it seldom makes a learned scholar; and if one be capable, twenty are not; and if the master sum up in brief the pith of his own long labours and travails, it is not easy to imagine that scholars can with less labour than it cost their masters dive into the depths thereof."

Here is the judgment of another of Oughtred's friends:[9]

". . . with the character I received from your and my

[8] Rigaud, *op. cit.,* Vol. I, pp. 16, 26. [9] Rigaud, *op. cit.,* Vol. I, p. 66.

noble friend Sir Charles Cavendish, then at Paris, of your second edition of the same piece, made me at my return into England speedily to get, and diligently peruse the same. Neither truly did I find my expectation deceived; having with admiration often considered how it was possible (even in the hardest things of geometry) to deliver so much matter in so few words, yet with such demonstrative clearness and perspicuity: and hath often put me in mind of learned Mersennus his judgment (since dead) of it, that there was more matter comprehended in that little book than in Diophantus, and all the ancients...."

Oughtred's own feeling was against diffuseness in text-book writing. In his revisions of his *Clavis* the original character of that book was not altered. In his reply to W. Robinson, Oughtred said:[10]

"... But my art for all such mathematical inventions I have set down in my Clavis Mathematica, which therefore in my title I say is tum logisticae cum analyticae adeoque totius mathematicae quasi clavis, which if any one of a mathematical genius will carefully study, (and indeed it must be carefully studied,) he will not admire others, but himself do wonders. But I (such is my tenuity) have enough fungi vice cotis, acutum reddere quae ferrum valet, exsors ipsa secundi, or like the touchstone, which being but a stone, base and little worth, can shew the excellence and riches of gold."

John Wallis held Oughtred's *Clavis* in high regard. When in correspondence with John Collins concerning plans for a new edition, Wallis wrote in 1666-67, six years after the death of Oughtred:[11]

"....But for the goodness of the book in itself, it is that (I confess) which I look upon as a very good book, and which doth in as little room deliver as much of the fundamental and useful part of geometry (as well as of

[10] *Ibid.*, p. 9. [11] *Ibid.*, p. 475.

arithmetic and algebra) as any book I know; and why it should not be now acceptable I do not see. It is true, that as in other things so in mathematics, fashions will daily alter, and that which Mr. Oughtred designed by great letters may be now by others be designed by small; but a mathematician will, with the same ease and advantage, understand A , and a^3 or *aaa*..... And the like I judge of Mr. Oughtred's Clavis, which I look upon (as those pieces of Vieta who first went in that way) as lasting books and classic authors in this kind; to which, notwithstanding, every day may make new additions.....

"But I confess, as to my own judgment, I am not for making the book bigger, because it is countrary to the design of it, being intended for a manual or contract; whereas comments, by enlarging it, do rather destroy it... But it was by him intended, in a small epitome, to give the substance of what is by others delivered in larger volumes...."

That there continued to be a group of students and teachers who desired a fuller exposition than is given by Oughtred is evident from the appearance, over fifty years after the first publication of the *Clavis*, of a booklet by Gilbert Clark, entitled *Oughtredus Explicatus,* London, 1682. A review of this appeared in the *Acta Eruditorum,* Leipsic, 1684, p. 168, wherein Oughtred is named "clarissimus Angliae mathematicus." John Collins wrote Wallis[12] in 1666-67 that Clark, "who lives with Sir Justinian Isham, within seven miles of Northampton," "intimates he wrote a comment on the *Clavis,* which lay long in the hands of a printer, by whom he was abused, meaning Leybourn."

We shall have occasion below to refer to Oughtred's inability to secure a copy of a noted Italian mathematical

[12] *Ibid.*, p. 471.

work published a few years before. In those days the condition of the book trade in England must have been somewhat extraordinary. Dr. J. W. L. Glaisher throws some light upon this subject.[13] He found in the Calendar of State Papers, Domestic Series, 1637, a petition to Archbishop Laud in which it is set forth that, when Hooganhuysen, a Dutchman, "heretofore complained of in the High Commission for importing books printed beyond the seas," had been bound "not to bring in any more," one Vlacq (the computer and publisher of logarithmic tables) "kept up the same agency and sold books in his stead."... "Vlacq is now preparing to go beyond the seas to avoid answering his late bringing over nine bales of books contrary to the decree of the Star Chamber." Judgment was passed that, "Considering the ill-consequence and scandal that would arise by strangers importing and venting in this kingdom books printed beyond the seas," certain importations be prohibited, and seized if brought over.

This want of easy intercommunication of results of scientific research in Oughtred's time is revealed in the following letter, written by Oughtred to Robert Keylway, in 1645:[14]

"I speak this the rather, and am induced to a better confidence of your performance, by reason of a geometric-analytical art of practice found out by one Cavalieri, an Italian, of which about three years since I received information by a letter from Paris, wherein was praelibated only a small taste thereof, yet so that I divine great enlargement of the bounds of the mathematical empire will ensue. I was then very desirous to see the author's own book while my spirits were more free and lightsome, but I could not get it in France. Since, being more stept into

[13] J. W. L. Glaisher, "On Early Logarithmic Tables, and their Calculators," *Philosophical Magazine*, 4th. Ser., Vol. XLV, 1873, pp. 378, 379.

[14] Rigaud, *op. cit.*, Vol. I, p. 65.

years, daunted and broken with the sufferings of these disastrous times, I must content myself to keep home, and not put out to any foreign discoveries."

It was in 1655, when Oughtred was about eighty years old, that John Wallis, the great forerunner of Newton in Great Britain, began to publish his great researches on the arithmetic of infinites. Oughtred rejoiced over the achievements of his former pupil. In 1655, Oughtred wrote John Wallis as follows:[15]

"I have with unspeakable delight, so far as my necessary business, the infirmness of my health, and the greatness of my age (approaching now to an end) would permit, perused your most learned papers, of several choice arguments, which you sent me: wherein I do first with thankfulness acknowledge to God, the Father of lights, the great light he hath given you; and next I congratulate you, even with admiration, the clearness and perspicacity of your understanding and genius, who have not only gone, but also opened a way into these profoundest mysteries of art, unknown and not thought of by the ancients. With which your mysterious inventions I am the more affected, because full twenty years ago, the learned patron of learning, Sir Charles Cavendish, shewed me a paper written, wherein were some few excellent new theorems, wrought by the way, as I suppose, of Cavalieri, which I wrought over again more agreeably to my way. The paper, wherein I wrought it, I shewed to many, whereof some took copies, but my own I cannot find. I mention it for this because I saw therein a light breaking out for the discovery of wonders to be revealed to mankind, in this last age of the world: which light I did salute as afar off, and now at a nearer distance embrace in your prosperous beginnings. Sir, that you are pleased to mention my name in your never dying papers, that is your noble favour to

[15] *Ibid.*, p. 87.

me, who can add nothing to your glory, but only my applause.....”

The last sentence has reference to Wallis's appreciative and eulogistic reference to Oughtred in the preface. It is of interest to secure the opinion of later English writers who knew Oughtred only through his books. John Locke wrote in his journal under the date, June 24, 1681, “the best algebra yet extant is Outred's.”[16] John Collins who is known in the history of mathematics chiefly through his very extensive correspondence with nearly all mathematicians of his day, was inclined to be more critical. He wrote Wallis[17] about 1667:

“It was not my intent to disparage the author, though I know many that did lightly esteem him when living, some whereof are at rest, as Mr. Foster and Mr. Gibson.....You grant the author is brief, and therefore obscure, and I say it is but a collection, which, if himself knew, he had done well to have quoted his authors, whereto the reader might have repaired. You do not like those words of Vieta in his theorems, ex adjunctione plano solidi, plus quadrato quadrati, etc., and think Mr. Oughtred the first that abridged those expressions by symbols; but I dissent, and tell you 'twas done before by Cataldus, Geysius, and Camillus Gloriosus[18] who in his first decade of exercises, (not the first tract), printed at Naples in 1627, which was four years before the first edition of the Clavis, proposeth this equation just as I here give it you, viz. $1ccc + 16qcc + 41qqc - 2304cc - 1836qc - 133000qq - 54505c + 3728q + 8064$ N *aequatur* 4608, finds N or a root of it to be 24, and composeth the whole out of it for proof, just in Mr. Oughtred's symbols and method. Catal-

[16] King's *Life of John Locke*, Vol. I, London, 1830, p. 227.

[17] Rigaud, *op. cit.*, Vol. I, pp. 477-480.

[18] *Exercitationum Mathematicarum Decas prima*, Naples, 1627, and probably Cataldus's *Transformatio Geometrica*, Bologna, 1612.

dus on Vieta came out fifteen years before, and I cannot quote that, as not having it by me."

"....And as for Mr. Oughtred's method of symbols, this I say of it; it may be proper for you as a commentator to follow it, but divers I know, men of inferior rank that have good skill in algebra, that neither use nor approve it.... Is not A^5 sooner wrote than A_{qc}? Let A be 2, the cube of 2 is 8, which squared is 64: one of the questions between Maghet Grisio and Gloriosus is whether $64 = A_{cc}$ or A_{qc}. The Cartesian method tells you it is A^6, and decides the doubt...."

There is some ground for the criticisms passed by Collins. To be sure, the first edition of the *Clavis* is dated 1631—six years before Descartes suggested the exponential notation which came to be adopted as the symbolism in our modern algebra. But the second edition of the *Clavis,* 1647, appeared ten years after Descartes's innovation. Had Oughtred seen fit to adopt the new exponential notation in 1647, the step would have been epoch-making in the teaching of algebra in England. We have seen no indication that Oughtred was familiar with Descartes's *Géométrie* of 1637.

The year preceding Oughtred's death Mr. John Twysden expressed himself as follows in the Preface to his *Miscellanies*:[19]

"It remains that I should adde something touching the beginning, and use of these Sciences.... I shall only, to their honours, name some of our own Nation yet living, who have happily laboured upon both stages. That succeeding ages may understand that in this of ours, there yet remained some who were neither ignorant of these Arts, as if they had held them vain, nor condemn them as superfluous. Amongst them all let Mr. William Ought-

[19] *Miscellanies: or Mathematical Lucubrations, of Mr. Samuel Foster, Sometime publike Professor of Astronomie in Gresham Colledge in London,* by John Twysden, London, 1659.

red, of Aeton, be named in the first place, a Person of venerable grey haires, and exemplary piety, who indeed exceeds all praise we can bestow upon him. Who by an easie method, and admirable Key, hath unlocked the hidden things of geometry. Who by an accurate Trigonometry and furniture of Instruments, hath inriched, as well geometry, as Astronomy. Let D. John Wallis, and D. Seth Ward, succeed in the next place, both famous Persons, and Doctors in Divinity, the one of geometry, the other of astronomy, Savilian Professors in the University of Oxford."

The astronomer Edmund Halley, in his preface to the 1694 English edition of the *Clavis,* speaks of this book as one of "so established a reputation, that it were needless to say anything thereof," though "the concise Brevity of the author is such, as in many places to need Explication, to render it Intelligible to the less knowing in Mathematical matters."

In closing this part of our monograph, we quote the testimony of Robert Boyle, the experimental physicist, as given May 8, 1647, in a letter to Mr. Hartlib:[20]

"The Englishing of, and additions to Oughtred's *Clavis mathematica* does much content me, I having formerly spent much study on the original of that algebra, which I have long since esteemed a much more instructive way of logic, than that of Aristotle."

WAS DESCARTES INDEBTED TO OUGHTRED?

This question first arose in the seventeenth century, when John Wallis of Oxford, in his *Algebra* (the English edition of 1685, and more particularly the Latin edition of 1693) raised the issue of Descartes's indebtedness to the English scientists, Thomas Harriot and William Oughtred.

[20] *The Works of the Honourable Robert Boyle in five volumes to which is prefixed the Life of the Author,* Vol. I, London, 1744, p. 24.

In discussing matters of priority between Harriot and Descartes, relating to the theory of equations, Wallis is generally held to have shown marked partiality to Harriot. Less attention has been given by historians of mathematics to Descartes's indebtedness to Oughtred. Yet this question is of importance in tracing Oughtred's influence upon his time.

On January 8, 1688-89, Samuel Morland addressed a letter of inquiry to John Wallis, containing a passage which we translate from the Latin:[21]

"Some time ago I read in the elegant and truly precious book that you have written on *Algebra,* about Descartes, this philosopher so extolled above all for having arrived at a very perfect system by his own powers, without the aid of others, this Descartes, I say, who has received in geometry very great light from our Oughtred and our Harriot, and has followed their track though he carefully suppressed their names. I stated this in a conversation with a professor in Utrecht (where I reside at present). He requested me to indicate to him the page-numbers in the two authors which justified this accusation. I admitted that I could not do so. The *Géométrie* of Descartes is not sufficiently familiar to me, although with Oughtred I am fairly familiar. I pray you therefore that you will assume this burden. Give me at least those references to passages of the two authors from the comparison of which the plagiarism by Descartes is the most striking."

Following Morland's letter in the *De algebra tractatus,* is printed Wallis's reply, dated March 12, 1688 (" Stilo Angliae"), which is, in part, as follows:

"I nowhere give him the name of a plagiarist; I would not appear so impolite. However this I say, the major part of his algebra (if not all) is found before him in other authors (notably in our Harriot) whom he does not

[21] The letter is printed in John Wallis's *De Algebra Tractatus,* 1693, p. 206.

designate by name. That algebra may be applied to geometry, and that it is in fact so applied, is nothing new. Passing the ancients in silence, we state that this has been done byVieta, Ghetaldi, Oughtred and others, before Descartes. They have resolved by algebra and specious arithmetic [literal arithmetic] many geometrical problems..... But the question is not as to application of algebra to geometry (a thing quite old), but of the Cartesian algebra considered by itself."

Wallis then indicates in the 1659 edition of Descartes's *Géométrie* where the subjects treated on the first six pages are found in the writings of earlier algebraists, particularly of Harriot and Oughtred. For example, what is found on the first page of Descartes, relating to addition, subtraction, multiplication, division and root extraction, is declared by Wallis to be drawn from Vieta, Ghetaldi and Oughtred.

It is true that Descartes makes no mention of modern writers, except once of Cardan. But it was not the purpose of Descartes to write a history of algebra. To be sure, references to such of his immediate predecessors as he had read would not have been out of place. Nevertheless Wallis fails to show that Descartes made illegitimate use of anything he may have seen in Harriot or Oughtred.

The first inquiry to be made is, did Descartes possess copies of the books of Harriot and Oughtred? It is only in recent time that this question has been answered as to Harriot. As to Oughtred it is still unanswered. It is now known that Descartes had seen Harriot's *Artis analyticae praxis* (1631). Descartes wrote a letter to Constantin Huygens in which he states that he is sending Harriot's book.[22]

An able discussion of the question, what effect, if any,

[22] See *La Correspondance de Descartes*, published by Charles Adam and Paul Tannery, Vol. II, Paris, 1898, pp. 456 and 457.

Oughtred's *Clavis mathematicae* of 1631 had upon Descartes's[23] *Géométrie* of 1637, is given by H. Bosmans in a recent article. According to Bosmans no evidence has been found that Descartes possessed a copy of Oughtred's book, or that he had examined it. Bosmans believes nevertheless that Descartes was influenced by the *Clavis*, either directly or indirectly. Says he:[24]

"If Descartes did not read it carefully, which is not proved, he was none the less well informed with regard to it. No one denies his intimate knowledge of the intellectual movement of his time. The *Clavis mathematica* enjoyed a rapid success. It is impossible that, at least indirectly, he did not know the more original ideas which it contained. Far from belittling Descartes, as I much desire to repeat, this rather makes him the greater."

We ourselves would hardly go as far as does Bosmans. Unless Descartes actually examined a copy of Oughtred it is not likely that he was influenced by Oughtred in appreciable degree. Book reviews were quite unknown in those days. No evidence has yet been adduced to show that Descartes obtained a knowledge of Oughtred by correspondence. A most striking feature about Oughtred's *Clavis* is its notation. No trace of the Englishman's symbolism has been pointed out in Descartes's *Géométrie* of 1637. Only six years intervened between the publication of the *Clavis* and the *Géométrie*. It took longer than this period for the *Clavis* to show evidence of its influence upon mathematical books published in *England*; it is not probable that *abroad* the contact was more immediate than at home. Our study of seventeenth century algebra has led us to the conviction that Oughtred deserves a higher place in the development of this science than is usually accorded

[23] H. Bosmans, S. J., "La première édition de la *Clavis Mathematica* d'Oughtred. Son influence sur la Géométrie de Descartes," in *Annales de la société scientifique de Bruxelles*, 35th year, 1910-1911, Part II, pp. 24-78.

[24] H. Bosmans, *loc. cit.*, p. 78.

to him; but that it took several decennia for his influence fully to develop.

THE SPREAD OF OUGHTRED'S NOTATIONS.

An idea of Oughtred's influence upon mathematical thought and teaching can be obtained from the spread of his symbolism. This study indicates that the adoption was not immediate. The earliest use that we have been able to find of Oughtred's notation for proportion, A.B:: C.D, occurs nineteen years after the *Clavis mathematica* of 1631. In 1650 John Kersey brought out in London an edition of Edmund Wingate's *Arithmetique made easie*, in which this notation is used. After this date publications employing it become frequent, some of them being the productions of pupils of Oughtred. We have seen it in Vincent Wing (1651),[25] Seth Ward (1653),[26] John Wallis (1655),[27] in "R. B.," a schoolmaster in Suffolk,[28] Samuel Foster (1659),[29] Jonas Moore (1660),[30] and Isaac Barrow (1657).[31] In the latter part of the seventeenth century Oughtred's notation, A.B:: C.D, became the prevalent, though not universal, notation in Great Britain. A tremendous impetus to their adoption was given by Seth Ward, Isaac Barrow, and particularly by John Wallis who was rising to international eminence as a mathematician.

In France we have noticed Oughtred's notation for proportion in Franciscus Dulaurens (1667),[32] J. Prestet

[25] Vincent Wing, *Harmonicon coeleste*, London, 1651, p. 5.

[26] Seth Ward, *In Ismaelis Bullialdi astronomiae philolaicae fundamenta inquisitio brevis*, Oxford, 1653, p. 7.

[27] John Wallis, *Elenchus geometriae Hobbianae*, Oxford, 1655, p. 48.

[28] *An Idea of Arithmetick, at first designed for the use of the Free Schoole at Thurlow in Suffolk....By R. B., Schoolmaster there*, London, 1655, p. 6.

[29] *The Miscellanies: or Mathematical Lucubrations, of Mr. Samuel Foster* ....by John Twysden, London, 1659, p. 1.

[30] *Moor's Arithmetick in two Books*, London, 1660, p. 89.

[31] Isaac Barrow, *Euclidis data*, Cambridge, 1657, p. 2.

[32] *Francisci Dulaurens Specima mathematica*, Paris, 1667, p. 1.

(1675),[33] R. P. Bernard Lamy (1684),[34] Ozanam (1691),[35] R. P. Petro Nicolas (1697).[36].

In the Netherlands we have noticed it in R. P. Bernard Lamy (1680),[37] and in an anonymous work of 1690.[38] In German and Italian works of the seventeenth century we have not seen Oughtred's notation for proportion.

In England a modified notation soon sprang up in which ratio was indicated by two dots instead of a single dot, thus A: B:: C: D. The reason for the change lies probably in the inclination to use the single dot to designate decimal fractions. W. W. Beman pointed out that this modified symbolism (:) for ratio is found as early as 1657 in the end of the trigonometric and logarithmic tables that were bound with Oughtred's *Trigonometria*.[39] It is not probable however that this notation was used by Oughtred himself. The *Trigonometria* proper has Oughtred's A.B :: C.D throughout. Moreover in the English edition of this trigonometry which appeared the same year, 1657, but subsequent to the Latin edition, the passages which contained the colon as the symbol for ratio, when not omitted, are recast, and the regular Oughtredian notation is introduced. In Oughtred's posthumous work, *Opuscula mathematica hactenus inedita*, 1677, the colon appears quite often but is most likely due to the editor of the book.

We have noticed that the notation A: B:: C: D antedates the year 1657. Vincent Wing, the astronomer, published in 1651 in London the *Harmonicon coeleste* in which is found not only Oughtred's notation A.B:: C.D but also

[33] *Elémens des mathématiques*, Paris, 1675, Preface signed "J. P."

[34] *Nouveaux élémens de géométrie*, Paris, 1692 (permission to print 1684).

[35] Ozanam, *Dictionnaire mathématique*, Paris, 1691, p. 12.

[36] Petro Nicolas, *De conchoidibus et cissoidibus exercitationes geometricae*, Toulouse, 1697, p. 17.

[37] R. P. Bernard Lamy, *Elémens des mathématiques*, Amsterdam, 1692 (permission to print 1680).

[38] *Nouveaux élémens de géométrie*, 2d. ed., The Hague, 1690, p. 304.

[39] W. W. Beman in *L'intermédiaire des mathématiciens*, Paris, Vol. IX, 1902, p. 229, question 2424.

the above modified form of it. The two are used interchangeably. His later works, the *Logistica astronomica* (1656), *Doctrina spherica* (1655) and *Doctrina theorica,* published in one volume in London, all use the symbols A: B:: C: D exclusively. The author of a book entitled, *An Idea of Arithmetick at first designed for the use of the Free Schoole at Thurlow in Suffolk...by R. B., Schoolmaster there,* London, 1655, writes A: *a*:: C: *c,* though part of the time he uses Oughtred's unmodified notation.

We can best indicate the trend in England by indicating the authors of the seventeenth century whom we have found using the notation A: B:: C: D and the authors of the eighteenth century whom we have found using A.B:: C.D. The former notation was the less common during the seventeenth but the more common during the eighteenth century. We have observed the symbols A: B :: C: D, (besides the authors already named) in John Collins (1659),[40] James Gregory (1663),[41] Christopher Wren (1668-69),[42] William Leybourn (1673),[43] William Sanders (1686),[44] John Hawkins (1684),[45] Joseph Raphson (1697),[46] E. Wells (1698)[47] and John Ward (1698).[48]

Of English eighteenth century authors the following still clung to the notation A.B:: C.D: John Harris's translation of F. Ignatius Gaston Pardies (1701),[49] George

[40] John Collins, *The Mariner's Plain Scale New Plain'd,* London, 1659, p. 25.

[41] James Gregory, *Optica promota,* London, 1663, pp. 19, 48.

[42] *Philosophical Transactions,* Vol. III, London, p. 868.

[43] William Leybourn, *The Line of Proportion,* London, 1673, p. 14.

[44] *Elementa geometriae....a Gulielmo Sanders,* Glasgow, 1686, p. 3.

[45] *Cocker's Decimal Arithmetick,....*perused by John Hawkins, London, 1695 (preface dated 1684), p. 41.

[46] Joseph Raphson, *Analysis Aequationum universalis,* London, 1697, p. 26.

[47] E. Wells, *Elementa arithmeticae numerosae et speciosae,* Oxford, 1698, p. 107.

[48] John Ward, *A Compendium of Algebra,* 2d ed., London, 1698, p. 62.

[49] *Plain Elements of Geometry and Plain Trigonometry,* London, 1701, p. 63.

Shelley (1704),[50] Sam Cobb (1709),[51] John Craig (1718),[52] Jo. Wilson (1724).[53] During the seventeenth century the notation A: B:: C: D acquired almost complete ascendancy in England.

In France Oughtred's unmodified notation A.B:: C.D, having been adopted later, was also discarded later than in England. An approximate idea of the situation appears from the following data. The notation A.B:: C.D was used by M. Carré (1700),[54] M. Guisnée (1705),[55] M. de Fontenelle (1727),[56] M. Varignon (1725),[57] M. Robillard (1753),[58] M. Sebastien le Clerc (1764),[59] Clairaut (1731),[60] M. L'Hopital (1781).[61]

In Italy Oughtred's modified notation $a:b::c:d$ found entrance the latter part of the eighteenth century. In Germany the symbolism $a:b=c:d$, suggested by Leibniz, found wider acceptance.[62]

It is evident from the data presented that Oughtred proposed his notation for ratio and proportion at a time

[50] George Shelley, *Wingate's Arithmetick*, London, 1704, p. 343.

[51] *A Synopsis of Algebra, Being a posthumous work of John Alexander of Bern, Swisserland....Done from the Latin* by Sam. Cobb, London, 1709, p. 16.

[52] John Craig, *De Calculo fluentium*, London, 1718, p. 35. The notation A: B:: C: D is given also.

[53] *Trigonometry*, 2d ed., Edinburgh, 1724, p. 11.

[54] *Méthode pour la mésure des surfaces, la dimension des solides..par M. Carré de l'académie r. des sciences*, 1700, p. 59.

[55] *Application de l'algèbre à la géométrie*... Paris, 1705.

[56] *Elémens de la géométrie de l'infini*, by M. de Fontenelle, Paris, 1727, p. 110.

[57] *Eclaircissemens sur l'analyse des infiniment petits*, by M. Varign, Paris, 1725, p. 87.

[58] *Application de la géométrie ordinaire et des calculs différentiel et intégral*, by M. Robillard, Paris, 1753.

[59] *Traité de géométrie théorique et pratique*, new ed., Paris, 1764, p. 15.

[60] *Recherches sur les courbes à double courbure*, Paris, 1731, p. 13.

[61] *Analyse des infiniment petits*, by the Marquis de L'Hopital. New ed. by M. Le Fèvre, Paris, 1781, p. 41. In this volume passages in fine print, probably supplied by the editor, contain the notation $a:b::c:d$; the parts in large type give Oughtred's original notation.

[62] The tendency during the eighteenth century is shown in part by the following data: *Jacobi Bernoulli Opera, Tomus primus*, Geneva, 1744, gives B.A :: D.C on page 368, the paper having been first published in 1688; on page 419 is given GE: AG = LA: ML, the paper having been first published in 1689. *Bernhardi Nieuwentiit analysis infinitorum*, Amsterdam, 1695, has on page 276,

when the need of a specific notation began to be generally felt, that his symbol for ratio $a.b$ was temporarily adopted in England and France but gave way in the eighteenth century to the symbol $a:b$, that Oughtred's symbol for proportion :: found almost universal adoption in England and France and was widely used in Italy, the Netherlands, the United States and to some extent in Germany; it has survived to the present time but is now being gradually displaced by the sign of equality =.

Oughtred's notation to express aggregation of terms has received little attention from historians but is nevertheless interesting. His books, as well as those of John Wallis, are full of parentheses but they are not used as symbols of aggregation in algebra; they are simply marks of punctuation for parenthetical clauses. We have seen that Oughtred writes $(a+b)^2$ and $\sqrt{a+b}$ thus, $Q:a+b:$, $\surd:a+b:$, or $Q:a+b$, $\surd:a+b$, using on rarer occasions a single dot in place of the colon. This notation did not originate with Oughtred but, in slightly modified form, occurs in writings from the Netherlands. In 1603 *C. Dibvadii in geometriam Evclidis demonstratio numeralis,* Leyden, contains many expressions of this sort,

$x:c-x::s:r$. Paul Halcken's *Deliciae mathematicae,* Hamburg, 1719, gives $a:b::c:d$. Johannis Baptistae Caraccioli, *Geometria algebraica universa,* Rome, 1759, p. 79 has $a.b::c.d$. *Delle corde ouverto fibre elastiche schediasmi fisico-matematici del conte Giordano Riccati,* Bologna, 1767, p. 65 gives $P:b::r:ds$. *"Produzioni mathematiche" del Conte Giulio Carlo di Fagnano,* Vol. I, Pesario, 1750, p. 193, has $a.b::c.d$. *Géométrie du compas,* by L. Mascheroni, translated by A. M. Carette, Paris, 1798, p. 188, gives $\surd 3:2::\surd 2:Lp$. Danielis Melandri and Paulli Frisi, *De theoria lunae commentarii,* Parma, 1769, p. 13, has $a:b::c:d$. *Institutiones analyticae,* Vicentio Riccato and Hieronymo Saladino, Vol. I, Bologna, 1765, p. 47, gives $x:a::m:n+m$. R. S. Boscovich, *Opera pertinentia ad opticam et astronomiam, Bassani,* 1785, p. 409, uses $a:b::c:d$. Jacob Bernoulli, *Ars Conjectandi,* Basel, 1713, has $n-r.n-1:c.d$. Pavlini Chelvicii, *Institutiones analyticae, editio post tertiam Romanum prima in Germania,* Vienna, 1761, p. 2, $a.b::c.d$. Christiani Wolfii, *Elementa matheseos universae,* Vol. III, Geneva, 1735, p. 63, has $AB:AE=1:q$. Johann Bernoulli, *Opera omnia,* Vol. I,Lausanne and Geneva, 1742, p. 43, has $a:b=c:d$. *Analyse des mesures des rapports et des angles,* by D. C. Walmesley, Paris, 1749, uses extensively $a.b::c.d$, later $a:b::c:d$. *Institutiones geometriae sublimoris,* by G. W. Krafft, Tübingen, 1753, p. 194, has $a:b=c:d$. J. H. Lambert, *Photometria,* 1760, p. 104, has $C:\pi=BC^2:MH^2$. *Meccanica sublime del Dott. Domenico Bartaloni,* Naples, 1765, has $a:b::c:d$. Occasionally ratio is not designated by $a.b$, nor by $a:b$, but by

$\sqrt{\cdot}\, 136 + \sqrt{2048}$, signifying $\sqrt{(136 + \sqrt{2048})}$. The dot is used to indicate that the root of the binomial (not of 136 alone) is called for. This notation is used extensively in *Ludolphi à Ceulen de circulo,* Leyden, 1619, and in *Willebrordi Snellii De circuli dimensione,* Leyden, 1621. In place of the single dot Oughtred used the colon (:), probably to avoid confusion with his notation for ratio. To avoid further possibility of uncertainty he usually placed the colon both before and after the algebraic expression under aggregation. This notation was adopted by John Wallis and Isaac Barrow. It is found in the writings of Descartes. Together with Vieta's horizontal bar, placed over two or more terms, it constituted the means used almost universally for denoting aggregation of terms in algebra. Before Oughtred the use of parentheses had been suggested by Clavius[63] and Girard.[64] The latter wrote for instance $\sqrt{(2 + \sqrt{3})}$. While parentheses never became popular in algebra before the time of Leibniz and the Bernoullis they were by no means lost sight of. We are able to point to the following authors who made use of them: I. Errard de Bar-le-Duc (1619),[65] Jacobo De Billy (1643),[66] one of whose books containing this notation was translated into English, and also the posthumous works of Samuel Foster.[67]

a, b, as for instance in A. de Moivre's *Doctrine of Chance,* London, 1756, p. 34, where he writes *a, b* :: 1, *q.* A further variation in the designation of ratio is found in James Atkinson's *Epitome of the Art of Navigation,* London, 1718, p. 24, namely, 3..2::72..48. Curious notations are given in Rich, Balam's *Algebra,* London, 1653.

[63] *Chr. Clavii Operum mathematicorum tomus secundus,* Mayence, 1611, algebra, p. 39.

[64] *Invention nouvelle en l'algèbre,* by Albert Girard, Amsterdam, 1629, p. 17.

[65] *La géométrie et pratique générale d'icelle, par I. Errard de Bar-le-Duc, Ingénieur ordinaire de sa Majesté.* 3d ed., revised by D. H. P. E. M., Paris, 1619, p. 216.

[66] *Novae geometriae clavis algebra, authore P. Jacobo de Billy,* Paris, 1643, p. 157; also an *Abridgement of the Precepts of Algebra. Written in French by James de Billy,* London, 1659, p. 346.

[67] *Miscellanies: or Mathematical Lucubrations, of Mr. Samuel Foster, sometime publike Professor of Astronomie in Gresham Colledge in London,* London, 1659, p. 7.

The symbol for the arithmetical difference between two numbers, $\sim$, is usually attributed to John Wallis but it occurs in Oughtred's *Clavis Mathematicae* of 1652, in the tract on *Elementi decimi Euclidis declaratio,* at an earlier date than in any of Wallis's books. As Wallis assisted in putting this edition through the press it is possible though not probable that the symbol was inserted by him. Were the symbol Wallis's, Oughtred would doubtless have referred to its origin in the preface. During the eighteenth century the symbol found its way into foreign texts even in far off Italy.[68] It is one of three symbols presumably invented by Oughtred and which are still used at the present time. The other two are $\times$ and ::

The curious and ill-chosen symbols, $\sqsubset$ for "greater than," and $\sqsupset$ for "less than," were certain to succumb in their struggle for existence against Harriot's admirably chosen $>$ and $<$. Yet such was the reputation of Oughtred that his symbols were used in England quite extensively during the seventeenth and beginning of the eighteenth centuries. Considerable confusion has existed among algebraists and also among historians as to what Oughtred's symbols really were. Particularly is this true of the sign for "less than" which is frequently written $\neg$. Oughtred's symbols, or these symbols turned about in some way, have been used by Seth Ward,[69] John Wallis,[70] Isaac Barrow,[71] John Kersey,[72] E. Wells,[73] John Hawkins,[74] Tho.

[68] Pietro Cossali, *Origine, trasporto in Italia primi progressi in essa dell' algebra, Vol. I, Parmense,* 1797, p. 52.

[69] *In. Is. Bullialdi astronomiae philolaicae fundamenta inquisitio brevis, Auctore Setho Wardo,* Oxford, 1653, p. 1.

[70] John Wallis, *Algebra,* London, 1685, p. 321, and in some of his other works. He makes greater use of Harriot's symbols.

[71] *Euclidis data,* 1657, p. 1; also *Euclidis elementorum libris XV,* London, 1659, p. 1.

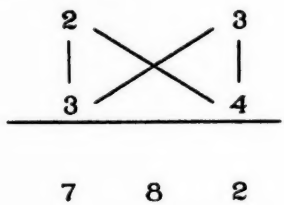
[72] John Kersey, *Algebra,* London, 1673, p. 321.

[73] E. Wells, *Elementa arithmeticae numerosae et speciosae,* Oxford, 1698, p. 142.

[74] Cocker's *Decimal Arithmetick,* perused by John Hawkins, London, 1695 (preface dated 1684), p. 278.

Baker,[75] Richard Sault,[76] Richard Rawlinson,[77] Franciscus Dulaurens,[78] James Milnes,[79] George Cheyne,[80] John Craig[81] and Jo. Wilson.[82]

General acceptance has been accorded to Oughtred's symbol $\times$. The first printed appearance of this symbol for multiplication in 1618 in the form of the letter x hardly explains its real origin. The author of the "Appendix" (be he Oughtred or some one else) may not have used the letter x at all but may have written the cross $\times$, called the St. Andrews cross, while the printer, in the absence of any type accurately representing that cross, may have substituted the letter x in its place. The hypothesis that the symbol $\times$ of multiplication owes its origin to the old habit of using two directed bars to indicate that two numbers are to be combined, as for instance in the multiplication of 23 and 34, thus,



has been advanced by two writers, C. Le Paige[83] and Gravelaar.[84] Bosmans is more inclined to the belief that

[75] Th. Baker, *The geometrical Key*, London, 1684, p. 15.

[76] Richard Sault, *A New Treatise of Algebra*, London (no date).

[77] Richard Rawlinson in a pamphlet without date, issued sometime between 16555 and 1668, containing trigonometric formulas. There is a copy in the British Museum.

[78] F. Dulaurens, *Specma mathematca*, Pars, 1667, p. 1.

[79] J. Milnes, *Sectionum conicarum elementa*, Oxford, 1702, p. 42.

[80] Cheyne, *Philosophical Principles of Natural Religion*, London, 1705, p. 55.

[81] J. Craig, *De calculo fluentium*, London, 1718, p. 86.

[82] Jo. Wilson, *Trigonometry*, 2d ed., Edinburgh, 1724, p. v.

[83] C. Le Paige, "Sur l'origine de certains signes d'opération," in *Annales de la société scientifique de Bruxelles*, 16th year, 1891-1892, Part II, pp. 79-82.

[84] Gravelaar, "Over den oorsprong van ons maaltecken ($\times$)," *Wiskundig Tijdschrift*, 6th year. We have not had access to this article.

Oughtred adopted the symbol somewhat arbitrarily, much as he did the numerous symbols in his *Elementi decimi Euclidis declaratio*.[85] In the absence of any further facts the mind is quite free to indulge in the sweets of unrestricted speculation as to the origin of this symbol.

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[85] H. Bosmans, *loc. cit.*, p. 40.

MUTATION CONCEPTS IN RELATION TO ORGANIC STRUCTURE.

THE MUTATION CONCEPT.

SINCE 1900, the idea of discontinuity, perhaps better termed definiteness, in variation, has steadily grown in biological circles. The classical experiments of de Vries on the discontinuous origin of characters in the evening primroses, and of Bateson and the Mendelians on the discontinuous inheritance of characters, has brought much greater definiteness and precision into the thinking on this subject. As a result of the experimental method the different kinds of variation can now be classified, though as yet imperfectly. The type of variation called by de Vries "mutation" has now been studied in many organisms —plants and animals—from bacteria to mammals; and the results of these studies make possible an analysis of the nature of mutation as a natural process.

Of all organisms, the Oenotheras among plants and the pumice fly, Drosophila, among animals, have been most intensively studied from this point of view. In recent years the cytological investigation of the Oenotheras[1] has thrown much light upon the nature of the mutation process, and has been particularly useful in limiting the speculations on

[1] See Gates, "A Study of Reduction in *Oenothera rubrinervis*," *Bot. Gaz.*, XLVI, 1-34, 1908. "Tetraploid Mutants and Chromosome Mechanisms, *Biol. Centralbl.*, XXXIII, 92-99, 113-150, 1913.

Gates and Thomas, "A Cytological Study of *Oenothera* mut. *Lata* and *Oe.* mut. ***semilata*** in Relation to Mutation," *Quart. Journ. Micro. Sci.*, LIX, 523-571, 1914, etc.

this subject. These cell studies combined with breeding experiments have not only shown that mutation is a phenomenon of variation and not merely of inheritance or hybridization, but they have also thrown considerable light upon the nature of the various changes involved in the origin of different mutants.

These questions have been considered in detail elsewhere,[2] but we may mention here a few of the conclusions regarding the nature of mutations, and a selection of the facts on which these conclusions rest. *Oenothera lata*, one of the mutants from *O. Lamarckiana*, has very characteristic foliage and habit, and is more or less completely male-sterile. This mutant was discovered by de Vries in 1887 and is now known to have constantly 15 instead of 14 chromosomes in its nuclei. This condition arises through one germ cell, when formed, receiving in addition to its normal number (7), a chromosome which does not belong to it. This extra chromosome therefore appears in the fertilized egg and is passed on by mitosis or cell division to every cell in the organism. The mutation is therefore a cell change propagated by mitosis, and the peculiarities of *lata* result from the fact that every nucleus contains an extra chromosome. We shall see that in the same way all the other mutations of *Oenothera* result from different kinds of cell change.

In contrast with *lata*, which arises through an irregular distribution of chromosomes in mitosis, we find in *gigas* a doubling of the whole series of chromosomes, to give an organism having 28 chromosomes in all its cells. The precise manner in which this condition, known as tetraploidy, arises is not yet clear, but a long list of cases of a similar relationship between related species is now known in many genera of plants. Thus we find it among the

[2] *The Mutation Factor in Evolution.* London: Macmillan, 1915. See especially Chapters IX and X.

violets, lady's tresses and cinquefoils, to mention only a few. the tetraploid condition is usually accompanied by gigantism not only of the cells but of the whole plant.

A third and very different type of mutation is found in *O. rubricalyx*, which originated from my cultures of *rubrinervis* in 1907. This mutant differs from its parent only in pigmentation. It is a marked color variety, having deep red buds, with red pigment developed also to a much greater extent in every part of the plant.[3] The offspring of this mutant gave *rubricalyx* and *rubrinervis* in the Mendelian ratio 3: 1. The chromosome-number in *rubricalyx* is unchanged—14, as in *rubrinervis* from which it originated. The origin and hereditary behavior of this mutant may therefore both be explained if we assume that one chromosome of *rubrinervis* has undergone a permanent chemical change so that its presence in the cell leads to greatly increased pigment-production. There is every evidence that this mutation is also a cell change, propagated throughout the developing organism by mitosis, for cells in all parts of *rubricalyx* show an increased content in the anthocyanin pigment.

With these few facts in mind we may now examine briefly some of the points of view which arise.

1. In order to be inherited completely the variation must arise in the nucleus of some cell or cells in the germ track of the organism. Wherever in the life cycle the change originally occurs (and this is usually during the process of chromosome reduction), it will come in the next generation to date from the fertilized egg. As the fertilized egg divides, the variation will be passed on as a *cell-feature* to all parts of the organism.

2. These nuclear changes are, as already observed, of various kinds, some of them essentially morphological,

[3] See Gates, "Studies on the Variability and Heritability of Pigmentation in Oenothera," *Zeitschr. f. Abst. u. Vererbungslehre*, IV, 337-372, 1911.

others essentially chemical. This conclusion is of much importance for evolution, for if correct it means that abundant material is supplied for evolution to have taken place in many directions at once. In this way the divergent and multifarious character of evolution is emphasized.

The Mendelian experimentation with hybrids has crystallized into the presence-absence hypothesis, according to which it is supposed that a recessive character is negative, due to the loss of something from the germ plasm, and a dominant character positive, due to the addition of something. Bateson[4] and others, basing their views of evolution on this hypothesis, have finally narrowed down the evolutionary process to the loss of "factors" or "inhibitors" for factors. This assumes that a single type of germinal change, such as occurs in the origin of *O. rubricalyx,* is the only type of discontinuous variation known. But we have already seen that mutation is a composite process, and that the various kinds of departures from the parent form cannot all be explained on the basis of one idea.

A still more fundamental alteration of the presence-absence hypothesis will be required if the view here expressed be correct. For this view implies that the origin of any pair of Mendelian characters is due, not to the loss or "dropping out" of something from the germ plasm of the organism, giving the negative (recessive) character, but to the sudden modification of the positive (dominant) character to produce the negative, or (in some instances) of the negative character to produce the positive. Thus in the case above mentioned, *O. rubricalyx* has originated from *rubrinervis,* not through the loss of an "inhibitor" from the latter but through a chemical alteration in the germ plasm, or rather in a particular part of it, namely, one chromosome. In the same way we may consider the differ-

[4] Bateson, *Problems of Genetics.* London: Humphrey Milford, 258 pages, 1913.

ence between a round pea and a wrinkled pea to have arisen through an alteration in the chemical nature of one chromosome of the round-pea race, of such nature that, when replacing the unchanged chromosome in the cell, it leads to the production of a race in which the sugar fails to be transformed to starch and the peas are therefore wrinkled. According to this view each new Mendelian character is to be looked upon as the result, not of the loss of something from the cell, but of a *modification* of one part or organ of the cell, or rather of the nucleus.

The conception that each recessive character has resulted from the loss of something from the germ plasm, was very reasonably founded on such cases as that of two white races of sweet pea which when crossed produce in F_1 the purple Sicilian sweet pea, the ancestor of both. It was very reasonable to suppose in such a case that each white race had arisen through the loss of a different "factor" for the production of color, since putting the two together by crossing immediately gave the color. And indeed, so long as no further evidence is available this method of viewing the matter is entirely legitimate and satisfactory.

But if we view each negative character as due to the *modification* rather than the loss of something, at the same time a similar result follows which is more in accord with a sound evolutionary view. Thus in the case of *Oenothera lata* we know that a change which is certainly not a loss, has occurred, yet in the cross *lata* × *nanella,* for example, *Lamarckiana* (the parent of both), as well as *lata* and *nanella,* are produced in F_1. In this instance, since *O. nanella* is a dwarf, it is easiest to think of it as having originated through the loss, or at least the permanent inactivity, of something in the germ plasm. Yet there is probably an equally fundamental sense in which even *nanella* represents a germinal modification rather than a loss,

while in *lata* the loss-conception is excluded. Hence, while the symbols of the presence-absence hypothesis are most useful in dealing with the inheritance of Mendelian characters, yet in order to be strictly accurate we need to modify this terminology when considering the *origin* of these character-differences.

3. We thus arrive at the position that not only is mutation a composite process involving various types of germinal change, but that Mendelian characters also originate in each case through the modification of a particular part of the germ cell. In the phylogeny of organisms we may therefore expect innumerable divergencies, with a number of lines sometimes radiating from the same point. The fact that parallel mutations, i. e., corresponding germinal changes in unrelated lines of phylogeny, frequently take place, increases this radiating tendency.

Aristotle, as is well known, arranged or attempted to arrange all organisms in a single linear series, and it is only in modern times, beginning with Lamarck, that the divergent and radiating nature of phylogenies have been recognized more and more. Herbert Spencer briefly traced the history of thought on this subject in his *Principles of Biology*. Since Spencer's time, as knowledge of the relationships of organisms has grown, this tendency has become still more pronounced, until now biologists recognize that probably never can three living species be arranged in linear series of descent, and paleontologists continually find that forms at first considered in the "direct line of descent" are rather on a longer or shorter "side line" of their own. All this is of course to be expected when we consider that particular elements of the germ plasm vary, each time, and not the germ plasm as a whole. The radiating tendencies must greatly outweigh any tendency to progress in a straight line. Mere divergence is of course to be expected on almost any hypothesis which assumes de-

scent with modification, as was abundantly shown by Darwin in the theory of natural selection. But the mutation conception as I have stated it, in which particular elements of the germ plasm vary independently each time, makes it possible much more easily to account for the prolific radiating tendencies which, according to the consensus of opinion of naturalists, organisms display.

THE EVOLUTIONISM OF BERGSON.

It seems worth while to compare the views of variation and phylogeny resulting from these experimental studies of mutations, with certain views on these matters expressed by Bergson.[5] I am not competent to discuss the fundamentals of Bergson's philosophy, and indeed that is unnecessary, for they have been criticised with admirable lucidity and, as it appears to me, with justice, by Bertrand Russell in this journal.[6] The critic points out, for example, that Bergson's views of number are confused and inconsistent and that number does not necessarily imply space; while Bergson's theory of duration and time rests upon a confusion between "the present occurrence of a recollection and the past occurrence which is recollected," i. e., between the act of knowing and the object known.

Zeno's argument of the arrow Bergson meets by denying that the arrow is ever at any point of its course. He criticizes the mathematical view of change by the statement that it "implies the absurd proposition that movement is made of immobilities." But Russell points out that this absurdity vanishes when it is realized that motion really implies change of relations. Finally the critic considers that when the identification of subject and object which results from Bergson's theory of duration is rejected, his whole system collapses, including the theories of time and

[5] *Creative Evolution.* New York, 1911.
[6] "The Philosophy of Bergson," *Monist,* XXII, 321-347, 1912.

space, his belief in real contingency, and his views that the universe contains only actions and changes but not things.

There are nevertheless certain elements in Bergson's views of evolution in their biological aspects, which appear worthy of consideration. Thus he has clearly recognized and emphasized the nature of the phylogenetic divergence to which I have referred. Life, in his view, is a tendency whose essence is to split up in the course of its development, giving simultaneous divergent or sometimes parallel lines of progress in the form of a sheaf. Such a separation of tendencies led to the plant world which specialized in fixity, insensibility and the accumulation and storage of energy from the sun by means of the chlorophyllian function; and to the animal world with mobility, consciousness, nerve centers and the explosive expenditure of energy in locomotion. All this may be regarded as sufficiently orthodox biology. The questions arise concerning the nature and causes of the diverging tendencies.

Bergson is also doubtless correct in pointing out (p. 135) as a cardinal error of philosophers from Aristotle onward, the idea that vegetative, instinctive and rational life are three successive degrees of development of one tendency. They are rather "three divergent directions of an activity that has split up as it grew," so that the difference is not one of degree but of kind.

Bergson believes that not only have organisms traveled over many diverging roads as an expression of the original *élan vital,* but they have constructed or created the road itself as they traveled; or rather, their own evolution constitutes the road, so that of necessity the road leads to no fixed or predetermined goal such as is implied in a teleological view. We may agree that at the present time science can offer no adequate reason for supposing that the particular directions many phylogenies have taken have been directly determined or narrowly limited by the

conditions on the earth's surface. The distinguishing features of, e. g., mosses, ferns, mammals, echinoderms and mollusks cannot be equated in terms of their different environments; indeed, each of the higher phyla of animals, such as reptiles and mammals, has tended to spread out and occupy all the main types of environment, aerial, terrestrial and aquatic. Nor can we formulate any adequate reason why the characteristic phyla of animals and plants should not, in another evolution under precisely the same earth-conditions, have worked out into quite different end-results. If this is the case, are we justified as biologists in taking the mechanistic attitude that all is given in the original protoplasm, and that the result of evolution has been in this sense a predetermined one? Bergson answers No, that the result has not been narrowly predetermined either in a mechanistic or a teleological sense; and I can see certain reasons for agreeing with him on this point, without at the same time becoming a vitalist. For it is not necessary to take the next step with Bergson, and assume that life is a force or impulse directed against matter and endeavoring to overcome its inertia. The biologist is perhaps more acustomed to think of life as a condition which appears when certain physical and chemical conditions of aggregation have been reached and disappears when the equilibrium of this system is sufficiently disturbed, — a mechanistic point of view.

And now to return to the reasons for agreeing with Bergson that evolution is to some extent, at any rate, indeterminate, so that the history of life on the earth under the conditions which have actually existed might have been different, as regards the characteristics of the various plant and animal phyla, from that which we have witnessed. This, I think, follows from the conclusion reached on a previous page—that particular elements of the germ plasm vary independently. For, this being the case, the *order* of

the occurrence of these variations will be more or less a matter of chance, depending in part on environmental vicissitudes; and each variation, like a move on a chess board, will help to determine by limitation the succeeding variations. If millions of different games can be played with thirty-two chess pieces, no limitation need be placed on the number of organic worlds which might have evolved from the first protoplasm. Nevertheless, if evolution has any meaning we see progress in complexity, and simultaneous development in many phyla as though a number of games were being played through simultaneously to the end without altering the rules of any game during its progress. The irritability of protoplasm and the increase in the organism's control over its environment are, however, rather slender threads on which to hang a completely mechanistic explanation of progressive evolution.

Many biologists will probably agree with Bergson when he says (p. 102), "The truth is that adaptation explains the sinuosities of the movement of evolution, but not its general directions, still less the movement itself." Nor should I dissent from the further statement (p. 101), "Evolution will thus prove to be something entirely different from a series of adaptations to circumstances, as mechanism claims; entirely different also from the realization of a plan of the whole, as maintained by the doctrine of finality. . . . It is one thing to recognize that outer circumstances are forces evolution must reckon with, another to claim that they are the directing causes of evolution." But although we may agree with these statements, we cannot admit the further one that evolution creates, as it goes on, "not only the forms of life, but the ideas that will enable the intellect to understand it." This leads to Bergson's peculiar and complicated philosophic views of the relations between the mind and things—that intellect and matter have progressively adapted themselves to one another in

the course of evolution (p. 204), a conception into which we need not enter.

Before leaving this aspect of the subject we may refer to Bergson's attitude toward the evolutionism of Spencer, which he criticizes as an attempt to reconstruct evolution from fragments of the evolved. His comparison of the Spencerian philosophy of evolution to the thought of a child which may put together the parts of a puzzle picture and in doing so imagine it is creating the design, is a telling argument for Bergson, who points out that the putting together of the fragments has nothing to do with the act of the original artist in producing the design.

We may, therefore, agree with Bergson that one of the great problems of evolution, so great that biologists have scarcely yet acquired the means of beginning a successful attack upon it, concerns the nature and causes of the general currents of animal and plant phylogeny. But his suggestion of an original impulse or *élan vital* impinging upon matter and spreading out into sheafs of organic movement, while a conception most stimulating to thought, is not of a sufficiently explanatory nature to be satisfying to the scientific mind.

VARIATION CONCEPTS IN RELATION TO ONTOGENY.

In his consideration of evolution Bergson naturally adopts attitudes, sometimes implied rather than definitely expressed, toward various questions of variation, heredity and ontogeny, which are worthy of discussion. Like many other writers he selects the eye as an example of a highly adapted organ, and asks how it can have arisen independently in mollusks and vertebrates through purely fortuitous circumstances. In stating the case, however, he is led to make certain assumptions which from our present point of view unnecessarily increase the admittedly great difficulties of any explanation. He says (p. 64): "If the

variations are accidental, how can they ever agree to arise in every part of the organ at the same time, in such way that the organ will continue to perform its function? Darwin quite understood this; it is one of the reasons why he regarded variation as insensible."

But I think we may say that we now know that inherited variations do *not* usually arise independently and simultaneously in different parts of the organism. According to the views of inherited variation already expressed in this paper, the inherited difference is due to a change in the fertilized egg. That change is a *single* thing, although in ontogeny it may work out in variations which express themselves in different parts of the organism and so appear to be independent of each other. Moreover, each inherited change is really a cell change, transmitted as such in plants through all the cells of the organism, and in animals, at least through the cell generations of the germ track. This is therefore the basis of the well-known cases of correlated changes which Bergson aptly characterizes (p. 66) as solidary, for instance the imperfection of the teeth in hairless dogs.

He finds greater difficulties with *complementary* changes, for example in different parts of the eye to improve its function. But these correlations are much less mysterious since the discovery of hormones, which may be produced by one organ and help to regulate the activity of wholly different and structurally remote and independent organs. Still more illuminating in this connection is the manner in which during development one organ is known to influence and even to cause the development of another. Thus it has been shown by Warren Harmon Lewis[7] that when a portion of skin from any part of the tadpole of a frog is grafted over the region where the

[7] Lewis, W. H., "Experimental Studies on the Development of the Eye in Amphibia," *Journ. Exptl. Zoology*, 1905, II, 431-446, pls. 2. Also *Amer. Journ. Anat.*, Vols. III, VI, and VII.

optic vesicle from the brain is developing, the skin of this region will invaginate and form a lens. From these experiments it may be concluded that in normal development the formation of the lens from ectoderm is the result of a stimulus emanating from the optic vesicle. With such facts in mind it is not difficult to understand that many changes apparently complementary and independent are really so interrelated as to be the result of a single change in the germ. The number of ways in which such interrelations can occur, increases the probability of explaining each case of correlated variations as the result of a single original change. This difficulty of Bergson's therefore disappears.

Pursuing the subject further, Bergson asks (p. 65): "How could the same small variations, incalculable in number, have ever occurred in the same order on two independent lines of evolution, if they were purely accidental? And how could they have been preserved by selection and accumulated in both cases, the same in the same order, when each of them, taken separately, was of no use?" The first question we will come to later. With regard to the second the case is overstated, because owing to the correlations of variations mentioned in the last paragraph, a *relatively* short series of evolutionary stages requires to be postulated to account for the evolution of the eye, though it may be admitted that a mere shortening of the series does not remove any of the difficulties. The second question harbors a misapprehension in supposing that the various stages in the perfecting of an organ are in themselves of no service to the organism.

Darwin was at some pains to show that the contrary is really the case. He held that each stage in the evolution of a particular structure was of use to its possessor, although the function of the structure might change completely in the process of development. Thus in the *Origin*

of Species (6th ed., I, 285-290), in answering an objection of Mivart, he suggests a comparison of the condition of the baleen in various whales with that of the palate in members of the duck family. He shows that in the ducks there is a series of stages in the development of the horny plates of the palate, yet each stage is serviceable to the species possessing it, though different species use the structure for different purposes. In conclusion Darwin remarks (p. 289): "Nor is there the least reason to doubt that each step in this scale might have been as serviceable to certain Cetaceans, with the functions of the parts slowly changing during the progress of development, as are the gradations in the beaks of the different existing members of the duck family." The important fact that apparently new organs are often a remoulding and readaptation of organs characteristic of members of a previous phylum or family, is frequently neglected. In such cases, as Darwin points out, the difficulty about the selection of undeveloped rudiments of an organ does not exist because the organ is functioning in every stage although the function changes gradually during the evolution. But the readaptation of organs can never account for their original appearance. This must have taken place in their simplest form, and it has generally been recognized that the fact of mutations or definite variations will help to bridge this difficulty.

Bergson turns next to the hypothesis of sudden variations, to see whether it will solve his problem. He says (p. 65): "It certainly lessens the difficulty on one point, but it makes it much worse on another. If the eye of the mollusk and that of the vertebrate have both been raised to their present form by a relatively small number of sudden leaps, I have less difficulty in understanding the resemblance of the two organs than if the resemblance were due to an incalculable number of infinitesimal resemblances acquired successively; in both cases it is chance that ope-

rates, but in the second case chance is not required to work the miracle it would have to perform in the first. . . . But here there arises another problem, no less formidable, viz., how do all the parts of the visual apparatus, suddenly changed, remain so well coordinated that the eye continues to exercise its function? For the change of one part alone will make vision impossible. . . . The parts must then all change at once, each consulting the others."

But we have already pointed out that the changes *must* be correlated and such as to make development and survival possible, for in order to be inherited they must have arisen in the egg, whence their influence radiates in ontogeny. Germinal changes leading to a less perfectly functional condition of the eye would immediately be eliminated by natural selection. Heredity maintains the species, while it "waits" for another germinal variation which will give the individual an advantage and so lead to the perpetuation of the change and its adoption by the species. The parts concerned do then "all change at once." This is not, however, because each consults the others but because all together are expressions of a germinal change which occurred in the egg. Nor is such a view teleological, because some changes will be advantageous, some innocuous, some bizarre, some harmful; but all unforeseeable even if we knew the nature of the change in the egg, for we at present know practically nothing of the relation between chemical composition and external form in organisms.

Bergson says truly (p. 67) that function is less narrowly bound to form in plants than in animals, a change in leaf-form for example producing no appreciable effect on the function of the leaf. In animals not only is there usually a closer relation between form and function, but there is also, as we have already observed, a greater interaction of organs upon one another through the blood circu-

lation, particularly by means of internal secretions and hormones.

Having briefly considered the subject of accidental variations, and found that Bergson's objections to chance variations as material for evolution are not always well founded, let us return to the subject of parallel evolution, as in the case of the vertebrate and the molluscan eye.

Patten[8] has studied the eyes of Pecten and of other mollusks and arthropods with much care, and we may therefore first examine his results. Some members of nearly all the higher invertebrate groups have relatively highly developed eyes. Thus they are found in many annelid worms and mollusks, and in nearly all Arthropoda. These eyes show a great variety of structure, but may be mainly classed as of three types: (1) the vertebrate type, having a lens; (2) much more commonly, the so-called "compound eye," composed of ommatidia and characteristic of the Arthropoda; and (3) much simpler structures composed essentially of depressed pigment spots. Very often, more than one type of eye occurs in the same individual, and there is no doubt that, as Patten says, "Many complex eyes have originated independently in very limited groups of animals." The same thing is true of various other structures, such as the seed in plants, the seed-habit having developed independently in different plant phyla. We agree with Bergson that the production of such organs represents the expression of a tendency; indeed, this is a frequent scientific form of statement of the facts as observed in phylogenies. The problem which Bergson fully apprehends and seeks to solve is, then, why do we find parallel expressions of the same tendency in independent phyla?

The tendency to form eyes does not appear in all phyla. Thus, so far as I am aware, definite eye structures do not

[8] Patten, Wm., "Eyes of Molluscs and Arthropods," *Mittheilungen zu d. Zool. Station zu Neapel,* VI, 542-756, pls. 28-32, 1886.

occur in any of the Echinodermata. One may think of this fact as indirectly connected with their mode of life, which does not call for highly developed vision. Useless characters which may have appeared through a single fortuitous mutation are to be found in all groups, particularly among plants. But an elaborate mechanism like the eye will only be developed when of sufficient advantage to the organism to have been built up by selection continuing in one direction for many generations. If vision is of no particular advantage to a starfish in its conditions of existence, then there is no "incentive" for the selection of any variations which may occur leading to the formation of organs having increased light-sensibility. Of course the absence of eyes in echinoderms may be due to some other cause, such as the very rudimentary condition of the nervous system, but this is at any rate a possible reason.

One of the remarkable features of the eyes in many mollusks in their great number. Thus, according to Patten, Arca has 250 compound eyes, 800 or 900 invaginated eyes or pigmented pits like those of Patella, and about 200 minute and simple ocelli, making a total of about 1300 eyes. And in addition to these there are numerous small groups of ommatidia. Again, in Pecten there are from 60 to 100 eyes of the highly developed type, while Onchidium and Chiton each have several thousand. In Pecten the eyes on the left mantle are usually arranged in pairs and are larger than those of the right mantle, the latter being spaced at regular intervals on long stalks. This is connected with the fact that the animal rests on its right valve and will turn over if placed on its left. In the young Pecten the ophthalmic fold of the mantle is covered with small pigmented pits like the invaginated eyes of Arca. There are also pigmented papillae containing a few ommatidia, but these later degenerate and disappear. Patten interprets this to mean that the ancestors of Pecten pos-

sessed many invaginated eyes and isolated ommatidia, a condition nearly comparable with that of the present Arca.

The structure of the larger eyes of Pecten is remarkably similar to that of the vertebrate eye. Without going into detail it may be mentioned that there is not only a lens but a cornea and pupil surrounded by a pigmented iris. Focal adjustment of the lens is accomplished (1) by change in shape of the lens, (2) by bodily movement of the lens by means of contractile fibres. It can be shown that a perfect inverted image of any object is thrown by the lens on the rods of the retina, the retina being inverted or reversed in structure as in the vertebrate eye. In certain species of Pecten a number of the eyes have their pupils covered with pigment so they must be functionless, yet they are perfect in structure.

In development, however, as Bergson points out (p. 75), the eye of Pecten differs from the vertebrate eye, in that the whole structure, including both the retina and the lens, is differentiated from an outgrowth from the mantle. Hence the lens does not arise from a separate invagination, and the retina is derived directly from the ectoderm.

Notwithstanding the remarkable efficiency of these molluscan eyes as a mechanism for forming images on the retina, yet the nervous system is in a rudimentary condition and the stimuli are carried to ganglia which cannot properly be considered a brain. In the vertebrates we may (if we choose) think of the image on the retina being transmitted along the optic nerve in the form of differences in nervous stimuli, much as a telephone wire conducts its current. From these stimuli-differences the brain reconstructs the "image" in consciousness. But these eyes in Pecten are evidently developed as image-formers, far beyond the possibility of their use by its simple ganglia. So conspicuous is this over-development that Patten tries to avoid the difficulty by supposing that the primary function of

the eyes in mollusks is to act as heliophags or absorbers of light-energy, while vision is a secondary function of the more highly developed eyes. But I know of no evidence for this view, and it does not seem to have been taken up later. Since the eyes are used in detecting shadows, to which the animal quickly responds, it is probable that this is sufficient to account for their high state of development. Bergson would probably say that the *élan vital* has impelled the organism to form eyes while leaving the nervous system rudimentary, but this is scarcely a causal explanation. On the other hand, one may suppose that natural selection was concerned in the development of these remarkable eyes, for each improvement in the eyes would render the organism better aware of its surroundings (even with an unimproved nervous system) and so aid in its preservation. For some unknown reason, favorable variations leading to great development of the nervous system have not occurred in Mollusca, and hence could not be selected. In the vertebrates on the other hand it is doubtless significant that the optic vesicle originates as a lateral outgrowth from the brain, which later controls the development of the lens. Hence in this case the evolution of the eye and the brain must have gone on together; for according to the view expressed in this paper, each successive germinal variation must have occurred so as to modify the rudiment from which develops both eye and brain. This being the case, as progressive variations successively appeared they would always be coordinated in the two structures, while retrogressive variations, being inefficient, would be eliminated.

Bergson stakes his whole case against mechanism on such instances as this of parallel evolution. Thus he says (p. 54): "Pure mechanism, then, would be refutable, and finality in the strict sense in which we understand it would be demonstrable in a certain aspect, if it could be proved

that life may manufacture the like apparatus by unlike means on divergent lines of evolution." But it is noteworthy that the philosopher and the scientist proceed from opposite ends of the story in approaching a subject of this kind. In discussing, for example, parallel adaptations, the scientist selects a simple case, and having offered an explanation proceeds to apply it to the more difficult ones. But M. Bergson always selects the most difficult and inscrutable instances in making it appear that science cannot solve the problem involved. The simpler cases he passes over entirely.

Thus we know that wings in one form or another have been evolved a number of times independently, as adaptations to aerial flight. The adaptation of a limb for aerial locomotion is a simple change compared with the development of the eye to react efficiently to the extremely delicate wave-motion of light. Yet one cannot but feel, as Darwin felt, that any explanation which applies to the simpler case must also be applicable to the more remote ones. We can understand the modification of fore-limbs into wings through the selection of definite variations which occurred in many directions but were of advantage for flight only when they occurred in one or a few directions. But in the more abstruse case of the eye it is possible to make the facts appear more recondite than they really are. For example, Bergson says (p. 71) regarding the eye: "Certainly the photograph [pigment spot in simple organisms] has been gradually turned into a photographic apparatus [eye]; but could light alone, a physical force, ever have provoked this change, and converted an impression left by it into a machine capable of using it?" In the first place, that the pigment spot is a direct response of the cell to light is an assumption which will require a good deal of proof. But if we consider the simpler case of the wing in flight, it seems absurd to ask, How could the air convert a limb into a struc-

ture capable of using the air in flight? We can realize that the air has not been acting on the limb to produce a wing, and neither need we assume that light acted on a pigment spot to produce an eye. The method has been much more round-about. Conclusions such as this of Bergson's lead us to feel that the solid ground of plodding science is preferable because safer than the more spectacular methods of philosophy.

It appears therefore that although we have not reached a complete understanding of the many cases of parallel evolution, yet the scientific method of attack is the only safe one to follow, for the philosopher's rapid strides constantly lead him into pitfalls.

THE RELATION OF INSTINCTS TO STRUCTURE.

The subject of the variation and development of instinct and intelligence contains several features of interest in this connection. Bergson considers that instinct and intelligence had a common origin, from which they evolved as diverging tendencies, — unlike solutions of the same problem,—culminating on the one hand in the ants and bees and on the other hand in man; the Arthropoda having specialized in instinct while the Vertebrata specialized in intelligence. These two psychic activities or methods of reaction to environment are therefore in a sense complementary and not consecutive stages in any evolutionary series, the most evolved intelligence still retaining something of instinct, and the most advanced instincts something of intelligence.

As in his treatment of parallel evolution, Bergson again singles out a few of the most striking and complex instincts when he comes to consider their origin. He particularly considers (p. 146) the beetle Sitaris which lays its eggs so that the larva will come in contact with the male bee, Anthophora, whence it passes to the female and thence to

one of her eggs where it undergoes a metamorphosis, feeding on the contents of the egg and afterwards on the honey in which the egg floats. The other instinct particularly considered (p. 172) is that of certain Hymenoptera (Ammophila, Scolia, Sphex) which sting their victims in the proper nerve centers to cause paralysis without death, and then store them up as food for the larvae when they hatch. Thus Sphex uses the cricket for this purpose, and stings each victim successively in its three ventral nerve ganglia so as to produce paralysis of movement.

It is, then, Bergson's aim to show that these instincts could not have been evolved by any of the methods proposed by science. He says (p. 169): "These instincts surely could not have attained all at once their present degree of complexity; they have probably evolved; but in a hypothesis like that of the neo-Darwinians the evolution of instinct could have come to pass only by the progressive addition of new pieces which in some way by happy accidents came to fit into the old. Now it is evident that in most cases instinct could not have perfected itself by simple accretion; each new piece really requires, if it is not to be spoiled, a complete recasting of the whole." And then he asks triumphantly, "How could mere chance work a recasting of the whole?"

But this all implies a mistaken conception of the relation of variation to ontogeny. The real variation, as we have already emphasized, in order to be inherited must arise in the germ cells, or the fertilized egg; and hence any variation, before it comes into expression, must have had an ontogenetic development which is a part of the ontogeny of the whole organism. Modification of an instinct, as of any other feature, through a variation, therefore means that *every ontogenetic stage is modified* and that the whole is necessarily to some extent recast. This reasoning applies most clearly to structural variations,

but the same reasoning must be true in its application to variations in instincts. If such variations have a structural basis at all (and how else can we think of them?) they must result from the unfolding of variations which occurred in the egg. Hence it is reversing the course of events to think of successive variations as "new pieces" which by "happy accidents" come to fit into the old. No such happy accident is required or indeed possible, for every variation-stage of each structure or instinct must be not only compatible with development but also with survival and inheritance, until a new variation (which is subject to the same limitations) again modifies not only the end-stage but the whole series of stages of ontogeny.

It should be pointed out that the same reasoning applies when species are modified through inheritance of acquired characters. For in this conception also the germ cells are modified and the modification expresses itself in the variation of the adult offspring; the only difference being that the neo-Lamarckian doctrine postulates the variation of the germ as resulting from the reflection of environmental effects from the soma back into the germ cells. In either case the modification of the germ cells expresses itself in a modified ontogeny and adult stage; but in one case the variation originated in the egg or sperm from unknown causes, while in the other it originates in the soma from stress of the environment, and secondarily affects the germ cells. The cases of what is known as parallel induction, in which the organism is environmentally modified in its ontogeny by new conditions, and at the same time its germ cells are so altered as to produce the modified type even under ordinary conditions,[9] are in harmony with these views.

Bergson goes on to say (p. 169): "I agree that an acci-

[9] Cf. W. E. Agar, "Transmission of Environmental Effects from Parent to Offspring in *Simocephalus vetulus*," *Phil. Trans. Roy. Soc.*, B, 1913, CCIII, 319-350, Fig. 5.

dental modification of the germ may be passed on hereditarily, and may somehow wait for fresh accidental modifications to come and complicate it. I agree also that natural selection may eliminate all those of the more complicated forms of instinct that are not fit to survive. Still, in order that the life of the instinct may evolve, complications fit to survive have to be produced. Now they will be produced only if, in certain cases, the addition of a new element brings about the correlative change of all the old elements. *No one will maintain that chance could perform such a miracle* (my italics); in one form or another we shall appeal to intelligence."

But we have already seen that an inherited variation *must* cause just such correlative changes. They cannot be avoided. This we regard as the teaching both of embryology and cytology in plants and animals. And if this happens with regard to structure we cannot see any reason why it should not happen in the evolution of instincts. Bergson here again raises difficulties which are really non-existent; but in this he has followed many biologists who have held a similar view. But if the view I have expressed be correct (and it appears to be the inevitable conclusion of the more recent cytological work in embryology and mutation), then the problem of adaptation is vastly simplified. It is no longer necessary to call in intelligence, as Bergson does, to account for the fact that "the addition of a new element" brings about the correlative modification of all the old elements. That is the natural and essential way in which all variations, whether in structure or instinct, become incorporated into the species.

The comparative study of instincts makes it clear that modifications in instinct and in structure go together, and it seems reasonable to suppose that such correlated variations result from a change in the structure of the egg. If this is the case, the need for an "effort" on the part of

the species, as Bergson suggests, to modify its instinct, is dispensed with. This idea of Bergson's is Lamarckism at its weakest.

It is no doubt a matter of great difficulty, if not impossibility, to conceive how certain instincts can be inherited, and therefore transmitted through the structure of the egg; e. g., the instincts already mentioned of the Hymenoptera which carefully sting their prey in the necessary spots to cause paralysis without death. But is the transmission of such an instinct through the structure of the egg any more difficult to conceive than the inheritance of intellectual differences in man, which we know to take place? If the instinct, like the structure, of the adult insect is implicit in the egg, then it is not necessary to invoke the inheritance of acquired characters, as does Bergson, to account for the origin and inheritance of instincts. The fact that instincts are variable does not militate against this view, for so are structures. May we not thus conceive of instincts developed stage by stage without stating them in terms of intelligence, though they are inherited, just as intelligence is inherited? This does not, however, help us to understand how instinct and intelligence are implicit in the structure of the egg.

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THE RELIGIOUS VIEWS OF EURIPIDES AS SHOWN IN THE "BACCHANALS."

THE "Bacchanals," apparently written by Euripides in his extreme old age and in Macedonia in an atmosphere wholly unlike that of Athens which he had left, contains religious views seemingly out of harmony with those expressed in his earlier dramas. And so, quite apart from the transcendent literary power of this magnificent play, "alone among extant Greek tragedies in picturesque splendor," so un-Greek in its enthusiastic proclaiming of man's affinity with nature, it has an importance in the development of religious thought in Greece. Its chief interest from this point of view is the long debated question whether it represents a recantation of earlier views of religion on the part of the aged poet.

At first glance the "Bacchanals" seems merely to record a phase of religious history—the victory of the late introduced cult of Dionysus into Greece. The wild and orgiastic rites of the primitive worship of the wine-god, though from the first exerting a powerful influence on the imagination of the people, could not have been accepted finally by the rational Greeks without great opposition. These exciting and secret rites, celebrated under cover of darkness and especially attracting the emotional natures of women, could never have gained their way in peace. The myth portrayed by Euripides, the persecution of the god in Thebes and his bloody revenge, seems to be but an echo of this

prehistoric conflict. This older worship, as Hartung remarks, "represents a return to the primitive condition of nature and a renunciation of civilization, that is, a renunciation of rational life regulated by morality and law and a return to the innocency of the wilderness. Hence the Maenads took fawns to their breasts and clad themselves in fawn-skins, to transform themselves, as it were, into roes; hence they crowned themselves with twigs of oak and fir, and ate raw flesh."[1]

It represents, therefore, a period long prior to the historic epoch when this crude worship had become metamorphosed and spiritualized by the great reform of Orphism, which spread over Greece and South Italy in the sixth century B. C. Thereafter it was no longer the religion of primitive men, who, like the barbarous Thracians, had worshiped animals as gods and had actually torn and devoured beasts of the mountains during their orgies when under the spell of the god. Euripides, in his brilliant tragedy, knows nothing of this spirit of reform, but pictures the wilder scenes of the earlier worship.

A closer examination of the play shows there must have been a deeper motive than merely painting, though in such glorious colors, the story of the early history of the newly revealed faith. For the choral odes, among the most beautiful of Greek tragedy, are all deeply religious in tone and constantly denounce rationalism, **τὸ σοφόν**, i. e., the subtleties of the current philosophical speculations which were undermining traditional beliefs. In proportion as such knowledge is depreciated, is faith in the established religion inculcated. I will quote from the play a few of the more notable sentiments in illustration of this. Thus the chorus asserts the divine providence and moral government of the world in the two following

[1] *Bakchen*, p. 156 (translated by Beckwith in his edition of the play, p. 10, n. 1).

passages, which would be difficult to parallel elsewhere in the poet's works:

"Verily the gods dwelling in the ether still can view the affairs of men. Human wisdom is oft no wisdom, nor is the thinking upon things which are unfit for mortal minds."[2]

"Slowly but surely the strength divine is roused and punishes those of mortal men who honor folly's ends, and, urged on by madness, do not extol what belongs to the immortals. For cunningly the gods conceal the lazy foot of time and hunt out at last the impious man. For 'tis no profit to learn nor practise beyond the stablished customs."[3]

This same acceptance of old beliefs is again urged in these words:

" 'Tis wise to keep both mind and heart from the lore of those who think themselves wise; whatever the common throng thinks and practises, that would I accept."[4]

Teiresias urges the same acceptance on Pentheus:

"Nor should we exalt mortal wisdom against that divine; our ancient traditions, which have existed from time immemorial, these no arguments shall overturn, nor the keenest subtleties of thought."[5]

The chorus praises the man who has renounced speculation, and who through the national faith has found knowledge of mysteries divine:

"Happy he who has 'scaped the storms of sea and reached his haven."[6]

"To preserve the mind in prudence and in a mood befitting mortals, brings a painless life to men who are ready to obey the behest of the gods."[7]

Human wisdom, however, is not to be neglected, but there are great mysteries beyond its ken:

"Wisdom I seek with diligence; but with joy I seek those other great things which direct our lives to what is good, both day and night teaching us to revere the gods and to throw aside all that violates the right."[8]

[2] 392 ff. The renderings are from my translation of the play published in *Records of the Past*, XI, Pt. 4, 1912.
[3] 882 ff. [4] 427 ff. [5] 200 ff. [6] 902 f. [7] 1002 ff. [8] 1005 ff.

From the consideration of such sentiments, we are almost persuaded into the belief that the play was written with the avowed intention of overthrowing the enemies of religion, and as an apotheosis of the popular faith. On this theory the play is merely what the Germans would call a *Tendenz-Drama,* and the moral is not hard to point. The inadequacy of human wisdom is shown in the character of Pentheus who, though well-intentioned, is a defender of τὸ σοφόν merely, and so is closed to all influences from that greater mysterious wisdom of the unknown whose glorification seems to be the chief purpose of the drama. His opposition, then, is but a signal example of rationalism failing to accept the supernatural, and he becomes but a type of the shallow free-thinker who, in accordance with his earth-born descent, has no insight into the mysteries of heaven, a type engendered by the sophistic teaching of Euripides's day, against which both the poet and Socrates strove. To quote Professor Moulton: "The plot of the play illustrates the unhappy fate of Pentheus, how those who oppose the worship of the vine are opposing a hidden omnipotence; if the votaries are imprisoned, an earthquake overturns the prison, chains drop off spontaneously, and a fire breaks out that men strive to quench in vain; or the Maenads themselves with supernatural might overturn trees and scatter the limbs of oxen with their hands."[9] It is just this contrast between the blindness of the Theban king, as seen in his scorn of the new superstition, and the hidden power of the god, which gives to the play its dramatic effect. His blindness drives him to madness and ultimately he rushes to his doom apparently with joy. Thus the whole intent of the drama appears to be didactic, that the acceptance of the national religion is the only true basis of human happiness and that the sceptical philosophy of the day is vicious and should be renounced.

[9] *Ancient Classical Drama,* p. 117.

Now in most of his other plays Euripides has nothing but contempt for the traditional theology.[10] This is most evident in the "Hippolytus," staged in 429 B. C., and in the equally powerful "Hercules Furens," which on grounds of style has been dated later, between 420 and 417 B. C. In the former the whole action of the plot is based upon a jealous feud between Aphrodite and Artemis. Because of a personal slight, the goddess of love inspires an ignoble passion in a mother for her step-son. Phaedra, who is one of the poet's noblest creations, Hippolytus and his father Theseus all become involved in an Olympian quarrel in which they are in no wise concerned. After the suicide of the mother and the destruction of the son, Euripides denounces the whole basis upon which the system of Olympus rests; for Artemis can not intercede for her favorite, though he has remained chaste, just because Aphrodite has willed it differently. Her own words are:

> "For Kypris willed that all this should befall
> To glut her spite, and this the Gods' wont is:—
> None doth presume to thwart the fixed design
> Willed by his fellow."[11]

In the "Hercules Furens" the poet also shows that the legendary imperfections of the gods evoked neither his faith nor praise. Here the plot turns on Hera's malicious persecution of her step-son and throughout the drama we are persuaded that she alone is to blame for the hero's madness and the consequent murder of his wife and children. When Hercules at last awakens from his frenzy and realizes his awful deed, he cries out in scorn:

> "To such a Goddess
> Who shall pray now?—who, for a woman's sake
> Jealous of Zeus, from Hellas cut off
> Her benefactors, guiltless though they were."[12]

[10] For his diatribe against the popular theology, see especially P. Decharme, *Euripide, et l'esprit de son théâtre* (Paris, 1893), Chap. 2; and cf. Verrall, *Euripides the Rationalist* (Cambridge, 1895), pp. 79-84.

[11] 1327 ff. (Way's translation).

[12] 1307 ff. (Way's translation).

Though the poet discloses the intriguing malevolence of the gods most clearly in these two plays, passages could be cited from all his other works in which they are held up to ridicule. Thus in the "Ion" Apollo is berated for lying and seduction;[13] in the "Andromache" the same god out of spite permits the murder of Neoptolemus, though the latter has come as a suppliant to his shrine.[14] In the "Electra"[15] and "Orestes,"[16] the murder of Clytemnestra, both its responsibility and consequences, is attributed to the Delphian god by Orestes, while in the "Iphigenia in Tauris"[17] the hero openly declares that Phoebus has deceived him. The injustice of the gods is a constant theme of the poet,[18] and the same iconoclastic spirit is seen in many of his dramas and fragments.[19] In fact only the "Alcestis" and "Suppliants" seem wholly free from such utterances, and Athena, Dionysus and Eros are about the only immortals who are left unscathed by the poet's profane hand. But it is unnecessary to quote further in evidence of his contempt of the received theology. In a word we can say that he never tries to soften the imperfections of the gods nor to bring out their higher natures as Æschylus and Sophocles did. They tried to "pour new wine into old bottles," to work over the old myths into harmony with their own sentiments by glossing over all that was objectionable. But Euripides, in view of his wide separation from traditional views, seemed to find such a reconciliation out of the question, and so, instead of trying to tone down their features, he brought out their grossness with perfect fidelity only that he might attack them the better. The famous fragment which runs

"If the gods do anything evil, then they are not gods,"[20]

[13] 436 ff. [14] 1111 ff. [15] 1190 ff. [16] 591 ff. [17] 711.

[18] E. g., "I. T.," 570 ff; "Troades," 469 f; "Herc. Fur.," 339 ff; "Cycl.," 355.

[19] E. g., "I. T.," 572 and 380 ff; "Hecuba," 488 ff; "Troades," 884 f; frags. 483, 793 etc.

[20] Frag. 294, 7. Cf. the sentiment in "I. T.," 391.

might be said to sum up all his objections to the traditional religion of his countrymen; in it we find the *Grundgedanke*—to quote Nestle[21]—of his whole polemic against Greek polytheism. And if we contrast this fragment with one of Sophocles, which asserts that "The gods never lead us into evil,"[22] we can gauge how essentially different was the point of view of these two contemporary poets. In one respect Greek drama was the gainer by Euripides's iconoclasm: the ideal which he failed to find in the gods he looked for in humanity. To him we are indebted for ideal types of mankind—such as Theseus for chivalry, Hippolytus for chastity, Alcestis for conjugal devotion, and many another. In many passages he even seems to take delight in contrasting the goodness of mortals with the capricious selfishness of the immortals.

If, then, we compare such iconoclastic sentiments as these with the perfervid religious ones of the "Bacchanals," the latter sound like a complete renunciation of speculative inquiry, like a retraction or "palinode" of earlier beliefs, or at least like an "eirenicon"—to use a phrase of James Adam[23]—or attempt of the poet to set himself right with public opinion before his death. The spirit of ethical contentment and speculative repose evident throughout the play seems to show that he was at last weary of his doubts and subtleties and that he had found peace in that same religion which he had denounced all his life. It would be a most striking example of poetic justice if this most skeptical of poets finally returned to the faith of his youth and met his end in conformity with Socrates's dictum "that a man should die in peace."[24] Accordingly, in spite of the fact that men of seventy and more do not so

[21] *Untersuchungen über d. philos. Quellen des Euripides*, 1902, p. 126; and cf. Gomperz, *Greek Thinkers*, II, p. 13.

[22] Frag. 226.

[23] *The Religious Teachers of Greece*, p. 312.

[24] Cf. "Phaedo," 117E.

easily change their lifelong opinions, this "recantation" explanation of the drama has been upheld by many able critics, e. g., Nägelsbach, Paley, Pohle, Wecklein, Bernhardy, K. O. Müller, Berlage, Pater and very recently Gomperz. Walter Pater has expressed it in these words: "Writing in old age, he is in that subdued mood,.... in which accustomed ideas, conformable to a sort of common sense regarding the unseen, oftentimes regain what they may have lost, in a man's allegiance. Euripides has said, or seemed to say, many things concerning Greek religion at variance with received opinion; and now, in the end of life, he desires to make his peace—what shall at any rate be peace with men. He is in a mood for acquiescence, or even for a palinode."[25]

However, this interpretation has had many equally strong opponents since Hartung first attacked it in 1844 in his *Euripides restitutus*, e. g., Roux, Patin, Bruhn, Nestle, Pfander, Tyrrel, Jebb, Decharme, Christ and Murray. The latter goes so far as to say that to look upon the play as "a reactionary manifesto in favor of orthodoxy is a view which hardly merits refutation."[26] In consequence, though many have been content merely to point out the vagueness and inconsistency of the poet,[27] others have offered very positive explanations of the purpose of the drama. The older view of Roux,[28] that the play is really a polemic against the popular faith, a thinly veiled criticism not only of the Dionysiac cult but of religion in general, has been revived in recent years.[29] Thus the scornful reply of Agave to Dionysus toward the end of the play,

[25] From his essay on the "Bacchanals," in *Greek Studies.*

[26] *Ancient Greek Literature,* p. 272.

[27] E. g., T. Rumpel, *De Euripidis atheismo,* Halle, 1839; J. Janske, *De philosophia Euripidis,* Breslau, 1857; and more recently Lewis Campbell, *Religion in Greek Literature,* London, 1898.

[28] E. Roux, *Du merveilleux dans la tragédie grecque,* Paris, 1846.

[29] E. g., by H. Patin, *Euripide,* Paris, 1894; E. Bruhn, in his edition of the *Bacchae,* Berlin, 1891.

"'Tis not meet that gods nurse their anger like men."[30]

has been taken as the starting-point for a reinterpretation of the piece on ironical grounds, on the theory that the poet spoke his own mind and meaning only in this one verse.[31] Similarly, the culminating motive of the play, Agave returning in triumph from her unwitting murder of her son, has been explained in an ironical light.[32] Still others[33] have discovered signs of malicious irony in the mystic legend of Dionysus contained in verses 286-297, though it has been pointed out often enough that the long speech of Teiresias in praise of the god as the giver of wine, inspirer of prophets and author of panics in armies (verses 266-327) could hardly have been interrupted by these verses which introduce a legend having nothing in common with the context. Consequently, most editors have rejected the passage,[34] and even if kept it should be looked upon merely as an account of the cult theology.[35] By a like process of reasoning the preceding speech of Pentheus (verses 242-7) has also been rejected. It is probable that the two passages in question were composed with reference to each other and added later. Other critics have found comic features in the character-drawing of Teiresias, especially in the passage in which the old seer describes Pentheus's madness (verses 200ff).[36]

[30] 1348.

[31] This is the view of Decharme, *op. cit.*; he is followed by H. Weil, *Etudes sur le drame antique*, Paris, 1897, and C. Lindskog, *Studien zum antiken Drama*, Lund, 1897.

[32] E. g., suggested by Campbell, *op. cit.*, pp. 309-310.

[33] E. g., G. Dalmeyda, *Ausgangspunkt der Bakchen*, Paris, 1908.

[34] It was kept, however, by both K. O. Müller, and Paley (*Eurip.*, Vol. II, p. 393). Dindorf rejected it because of its "*dictio inepta confusa omninoque non Euripidea*," and because it interrupts the context; he is followed by Tyrrel and others.

[35] As the following have done: Weil, *Etudes*, p. 113f; R. Hirzel, *Berichte der sächs. Gesell. der Wiss.*, XLVIII, 1896, p. 294; Christ, *Gesch. der griech. Litt.*, 6th ed., 1912, p. 374, n. 4.

[36] E. g., P. Girard, *Rev. des études grecques*, XVII, 1904, pp. 175f, in connection with a fantastic attempt to join the three plays which were brought out in the year after the death of Euripides (the "Iphigenia in Aulis," "Alcmaeon"

But is the real intention of the "Bacchanals" to be explained on either the "recantation" or "irony" theory? As for the latter, it may be said at once that with the exception of the line in question (1348) there is not a single other indication in the play of any of the blasphemous expressions which are so common in the poet's other works, and that instead of betraying any criticism of the wine-god or his cult, the entire drama extols his might with the utmost warmth and vigor. In fact it is extolled with such seriousness that Euripides cannot be said to give an unbiased account of the struggle between Pentheus and the hidden power of the god. For though his human sympathies are certainly with the unhappy upholder of "reason," just as they were with Hippolytus in the earlier play, still he seems to be wholly within the influence of the religious antagonism to reason, and to be writing as a subjective believer in the religious views expressed. Of course a good deal of this attitude can be explained by his desire to give a powerful and effective stage setting to the play.

On the other hand the advocates of the "palinode" theory are obliged to assume an essential change in the poet's attitude toward religion, and so to look upon the "Bacchanals" as a sort of death-bed confession of earlier heresies. But are we justified in assuming any such repentant relapse of the aged poet into the old epic orthodoxy which he had impugned all his life long? This was the view of Bernhardy[37] and Nägelsbach[38] long ago and has been recently revived by Gomperz,[39] who thinks that in this way the poet wished to atone for his "*Abfall vom Genius seines Volkes.*"

and "Bacchanals") in a *trilogie libre*. His "comic" theory has found adherents in Dalmeyda (*op. cit.*) and O. Schröder, *Zeitschr. für Gymnasialwesen*, LXIV, 1910, p. 193.

[37] *Griech. Litt.*, II, 2.

[38] *Nachhomerische Theologie*, Nürnberg, 1857, p. 463ff.

[39] *Griech. Denker*, II, 12.

Though the utterances of the chorus and the fate of Pentheus protest that speculative philosophy must be renounced, still it is perfectly clear that the poet's conception of Dionysus is rationalistic, and that he is pictured as in no sense a personal god. As Gilbert Murray has said: "If Dionysus is a personal god at all, he is a devil."[40] On any such theory the whole moral purpose of the play would be vitiated. But the god is nothing more than a personified principle, a rationalized idea, like the conception of Aphrodite in the "Hippolytus." Thus Teiresias, in his effort to convert Pentheus, says:

> "But two things, oh youth, find worth among mankind; first our goddess Demeter; for she is earth, call her by what name thou wilt; 'tis she who nourishes men with food; but now Semele's offspring hath given us that liquid strength hidden in the grape, a boon to men, for it assuages the grief of wretched mortals so soon as they are filled with the sweetness of the vine; and it grants sleep, oblivious of daily toil, for forgetfulness is their only cure; and this gift of Bacchus is poured out in libation to the gods and through its means men are blessed."[41]

We should remember that the Theban seer always speaks with authority in Greek tragedy and is generally the mouthpiece of the dramatist, and so this rationalistic conception of Dionysus was doubtless Euripides's own. The sophist Prodicus had already conceived of Dionysus as the apotheosis of wine and Demeter of corn, and had identified Poseidon with water, Hephaestus with fire, etc. Cicero —who quotes his teaching in the *De natura deorum,* 1, 118—looked upon this personification of the gods as natural objects as a complete denial of religion. So Euripides looked upon Dionysus merely as a principle—the embodiment of enthusiasm, not only the god of wine, but, in Adam's words, "a higher personification of passion in religion and joy in life,"—such a principle as that

[40] *Anc. Greek Lit.,* p. 272. [41] 274ff.

described by Plato in the "Phaedrus," where Socrates, in distinguishing good from evil madness, mentions four kinds of the former, the third of which he calls "poetic" madness. He says it takes "hold of a delicate and virgin soul, and there inspiring frenzy, awakens lyrical and all other numbers."[42] This poetic madness is best illustrated by the "Bacchanals," though there are indications in the play of the other varieties of madness as well. As Adam has observed, there is no other Greek poem in which the writer is so "possessed."[43]

Though Euripides was no consistent follower of Orphism, still he was interested in its mystic and ascetic phases, as we know from the fact that he devoted at least one play—the lost "Cretans"—to this subject. The two gods or "principles" of that sect, Dionysus and Eros, were always reverently treated in his plays. It may be that the early associations of his birthplace Phlye in Attica, where mysteries were celebrated in honor of Demeter and Core as well as Eros, the cosmic spirit of Orphism, influenced his attitude toward mysticism, just as Æschylus was influenced by the mysteries celebrated at his birthplace Eleusis. These gods, Dionysus and Eros, were nothing but "potencies" to the Orphics. As Miss Harrison has said:

"The religion of Orpheus is religious in the sense that it is the worship of the real mysteries of life, of potencies (δαίμονες) rather than personal gods (θεοί); it is the worship of life itself in its supreme mysteries of ecstacy and love.... In ancient Greek religion these (Bacchus and Eros) are the only real gods. Orpheus dimly divined the truth later to become explicit through Euripides....It is these real gods, this life itself, that the Greeks, like most men, were inwardly afraid to recognize and face, afraid even to worship....Now and again a philosopher or poet, in the very spirit of Orpheus, proclaims these true gods,

[42] 245 (Jowett). [43] *Op. cit.*, p. 315.

and asks in wonder why to their shrines is brought no sacrifice."[44]

Since, then, Dionysus in the "Bacchanals" is conceived merely as the rationalized principle of enthusiasm, it is clear that the main problem of the play is not a question of skepticism against orthodoxy, but the relative value of "reason" and "enthusiasm" in life.[45] So the whole purport of the play is epitomized in such utterances as these which have already been quoted: "Human wisdom is oft no wisdom"; "Nor should we exalt mortal wisdom against that divine." Rationalism is denounced as insufficient; human knowledge, though valuable, is infinitesimally small in comparison with the great mysterious knowledge beyond, but yet it must not be neglected; "Wisdom I seek with diligence (τὸ σοφόν οὐ φθονῶ); but with joy I seek those other great things which direct our lives to what is good." We must bear in mind that the rationalism which the poet here condemns is only that of the sophists, the same which he had condemned long before in the "Hippolytus" and "Medea." There is something greater than this, and that is religious exaltation, which he offers as the true wisdom. As Gilbert Murray says: "Reason is great but it is not everything. There are in the world things not of reason, but both below and above it; causes of emotion, which we cannot express, which we tend to worship, which we feel perhaps to be the precious elements in life. These things are gods or forms of God; not fabulous immortal men, but 'Things which are,' things utterly non-human and non-moral, which bring man bliss or tear his life to shreds without a break in their own serenity."[46] He goes on to say that this is the kind of religion against which

[44] *Prolegomena to Greek Religion*, p. 658. In the "Symposium" of Plato, 189, Aristophanes says mankind has never understood the power of Eros, else they would have built him great shrines. In the "Hippolytus," the chorus sings a similar refrain, 538ff.

[45] Cf. Adam, *op. cit.*, p. 316, whom I follow in this connection.

[46] *Anc. Greek Lit.*, p. 272.

Tolstoy preached, which Bentham and Paley tried to abolish, and which Plato denounced and followed. And in a more recent work the same writer has given the rational basis of the Dionysiac worship in these words: "Dionysus was a fiction, the ritual of Dionysus a reality—the reality in fact out of which the fiction was developed or projected. It is the ritual of the spring, of the New Year, of *le renouveau*—the renewal after the dead winter of all the life of the world. . . . Further, if we would understand Dionysus-worship, we must realize that these vegetation-cults and all their grossness were bound up with the things that are the most beautiful in the world."[47] This true basis of the Dionysiac worship as the negation of rationality was felt by Euripides when writing this play, which pictures so passionately the sympathy of mankind with nature.

From this view-point of the rationalistic conception of the wine-god, the question as to whether the god or one of his priests played the chief rôle in the drama does not have the importance which many have thought. Doubtless the gap in the one manuscript preserving the play—ending with verse 1330, in which Dionysus continues his prophecy from the *theologeion*—would, if recovered, settle the question finally as to whether the Lydian stranger was Dionysus or a preacher of the new religion. However rationalistic Euripides's conception of Dionysus was, we should not forget that to an Athenian audience Dionysus was a personal god. Now he is portrayed as a human character throughout the action of the play; and to avoid the shearing and binding scene, which would have appeared repugnant in a theater whose representations were merely acts of homage to the god, we are justified in looking upon the comely stranger as a priest or an adept of the common type, inspired, if you like, to perform miracles which the ordinary stage-machinery of the day could have easily

[47] From *Greek and English Tragedy: A Contrast.*

represented. Nor are we obliged to explain features in the action of the play,—as, e. g., the destruction of the palace —on any theory of hypnotism of the audience as has been attempted recently.[48] Thus, it is more reasonable to assume that the god appeared only in the prologue and epilogue as is customary in the dramas of Euripides. But the loss of the passage in question is irreparable from a wholly different cause. We have lost almost the whole of Agave's speech, for it breaks off after the first verse (1329). The scene represented a frenzied mother who had unconsciously slain her son and transfixed his head on a spear; she slowly recovers her sanity and laments over the dead body. In so heart-rending a scene as this, the poet must have furnished a perfect example of Aristotle's idea of a "recognition."[49] If we compare the sentiment of this lost speech with that of Hecuba wailing over the body of Astyanax as preserved in the "Troades," we can imagine how effectively this most sympathetic of poets must have rendered so terrible a situation, perhaps the most moving he ever wrote, and we can thus form some idea of our loss.[50]

So if it was the intention of Euripides in the "Bacchanals" to portray Dionysus as a rational principle and thus to denounce the pretensions of a false philosophy as inadequate, the play is in no sense a reaction toward dogmatic orthodoxy. A study of his other dramas shows that though he was a disbeliever in the traditional theology, he had never actually denied the essential basis of religion. Though "by nature a destroyer of illusions,"[51] he probably

[48] As in G. Norwood's *Riddle of the Bacchae*, 1908.

[49] "Poetics," 1454a[2].

[50] Two passages in Apsines, a writer on rhetoric (*Rhet. Gr.*, IX, pp. 587 and 590, ed. Walz) gives us a faint idea of the purport of the speech. The author of *Christus Patiens*—wrongly ascribed to Gregory of Nazianzus—also probably had the missing portion before him. Hartung (*Euripides restitutus*) and Kirchhoff (*Philol.*, VIII, pp. 78-93) have reconstructed several lines of the lost passage of the "Bacchanals" from that drama.

[51] Croisèt (*Hist. de la littérature grecque*, I, p. 313) says of him: *"C'était par nature un destructeur d'illusions."*

never expressed disbelief in the idea of deity. It is true that his fellow Athenians looked upon him as a free-thinker; Aristophanes has testified to their opinion in a famous passage in the "Thesmophoriazusae" in which a poor widow accuses the poet of depriving her of her livelihood—she was a weaver of sacrificial chaplets—by his teaching that there are no gods.[52] And the more famous fragment from the lost "Bellerophon," preserved to us by Justin Martyr,[53]

> "Doth any say there are Gods in heaven?
> Nay, there are none,"

has also been urged to prove the poet's disbelief in deity. But we have no idea of the context in which this fragment occurred; if the succeeding line were preserved, quite possibly its whole meaning would be different. We must be on our guard against accepting such fragments as conclusive evidence because of their disjointed nature and the fact that they are often tinged with Christian or Jewish interpolations.[54] It is always perplexity and doubt rather than positive disbelief which are the burden of many another passage. Thus in the "Helena" the chorus complains that no one can tell

> "What is God, or what is not God, or that which lies between."[55]

In the "Hercules Furens" the famous doubt is expressed:

> "Zeus, whoever Zeus is."[56]

And the same agnosticism meets us in this line of the "Orestes":

[52] 445ff.

[53] Frag. 288; see his "De Monarchia," ch. 5.

[54] Tyrrell (in his edition of the *Bacchae*, p. XXIII) cites frag. 256 (from the lost "Achelaus," which probably reflected a similar mental state as the "Bacchanals") as in itself evidencing the poet's dissatisfaction with the moral government of the world, a sentiment fortunately condemned in the preserved answer. He also notes that frag. 852 is of Jewish or Christian origin.

[55] 1327.

[56] 1263f, repeated in frag. 483. Adam (p. 444) remarks that "Greek writers not infrequently represent the Highest God as the inscrutable one whose name is not lightly to be spoken" and cites the "Troades," 885, and Plato's "Euthyphron," 12A, as examples.

"In thraldom to the Gods we live, whoever the Gods may be."[57]

He seemed to be constantly thwarted by the obscurity of everything connected with theology. Orestes's remark in the *Iphigenia in Tauris,*

"In things divine great confusion reigns,"[58]

might be quoted as a summary of his doubts. That there is no way, either by divination or otherwise, to learn the will of the gods whose purpose is ever invisible to man, is a common sentiment of the poet.[59] Iphigenia says:

"For all the acts of the gods move on invisibly and no one knows anything clearly."[60]

But such isolated passages—many of which are put into the mouths of actors only to be denounced — can only be interpreted in the light of his teaching as a whole. His polemic was aimed against the anthropomorphic ideas of the gods held by his countrymen, and so he came to be looked upon as an atheist. Whether he believed in ideal gods it is hard to determine, for here the evidence is again vacillating; sometimes he seems to believe and sometimes not. But that he had arrived at certain definite assumptions as to the true nature of the godhead, whose various names—Ether, Law, Necessity, Justice, Reason—are but attributes of the one all-embracing infinite substance, can be shown by adducing many passages in which the poet maintains that the gods must furnish a moral standard for men, that the notions of hegemony and clashing wills implied in polytheism are wrong and that the divine nature is self-sufficient.[61] Iphigenia cannot believe that Artemis really enjoys human sacrifices, but argues that the barbarous Taurians have attributed their own murderous customs to the goddess, and finally says:

[57] 418. [58] 572.

[59] E. g., "H. F.," 62; "Hel.," 744-5; "Alc.," 785f.

[60] "I. T.," 476-7; cf. Solon, frag. 16 (Hiller).

[61] E. g., "H. F.," 1307ff; and 1342ff; "I. T.," 385ff; "Ion," 442ff; frag. 1130, etc.

"None of the gods I ween is evil or doeth wrong,"[62]

a sentiment which can be looked upon as the central idea in Euripides's constructive theology. That he believed that justice would finally guide all things to their goal is clear from the beautiful prayer which Hecuba makes to Zeus in the "Troades," ending:

"....for treading soundless paths
In justice dost thou guide all mortal things."[63]

Thus Euripides did not reject the basic facts of religion but tried to interpret them in a way which would be in harmony with a belief in the benevolence of the divine nature.

Though the theology of the "Bacchanals," then, is conceived mainly in the same rational spirit which we see in his earlier works, still no one can read the play without being convinced that some great change has come over the religious attitude of the poet, who with such youthful fire and in such passionate language thus proclaims the power of this irresistible world-embracing divinity, which differs so essentially from the old Olympian gods. For this change[64] I think we should look almost wholly to the circumstances of the composition of the play in Macedonia, the home of the Dionysiac cult. In writing for a Macedonian audience it was but fitting that Euripides should have chosen for his subject the worship of their great god. Frequent allusions to the country clearly show his desire to compliment his friend and host Archelaus.[65] And the wholly un-Greek character of the play—only matched, perhaps, among his other works by the "Orestes"—was far better suited to the genius of this land of orgiastic worship

[62] "I. T.," 391; cf. the same thought in "Troades," 987ff.

[63] 887-8 (Way).

[64] Against the view of, e. g., Decharme, Weil, Tyrrell, Nestle (*Philol.*, 58, 1899, p. 362f) and most radically, Lindskog; the change is assumed by Christ and others.

[65] E. g., Pieria, 410 and 565; "Olympus," 561; the "Axius," 569, and "Lydias," 571.

and mystic ceremonies than to the more temperate states of Greece. The old poet, weary of his logical subtleties and lifelong doubts, has finally found peace in a form of mysticism—the mystic worship of Dionysus, whose real nature was first made clear to him here far from Athens, where he is now breathing an atmosphere of intellectual freedom. As Jebb has said: "The really striking thing in the 'Bacchae' is the spirit of contentment and of composure which it breathes,—as if the poet had ceased to be vexed by the seeming contradictions which had troubled him before."[66] The tendency toward mysticism,[67] long dormant in him, has at length asserted its power and now has full reign. He has finally, contrary to his custom, adopted the spirit of an enthusiastic convert; we are persuaded that he is convinced of all that he so passionately writes; the entire drama is pervaded with the exaltation of an overpowering vision.[68] Dominated by the new enthusiasm, he has returned to the peaceful worship of nature and no longer lets his feelings be restrained by any ethical or reflective doubts. James Adam has finely said: "No other ancient poem shows so rapturous a feeling of the kinship between man and nature. The very hills are thrilled with ecstacy in sympathy with the frenzied votaries of the god.[69] We feel that Dionysus has become a power pulsating throughout the whole of nature, both inorganic and organic, making the universe into a living, breathing whole; and we are stirred with a new sense of unification with the mystery

[66] *Encycl. Brit.* (11th ed.), art. "Euripides."

[67] Christ (*op. cit.*, p. 375) has found traces of this mystic tendency in his earlier dramas, e. g., in the "Ion," where the mystical renunciation of the world is glorified; in the character drawing of Eteocles in the "Phoenissae"; and in the "Cyclops," where rationalism is exposed. Recently Gomperz (II, p. 15) thinks he sees the same attitude of mind in the "Hippolytus."

[68] He is so much in sympathy with his subject that some have argued that he merely intended to terrorize his audience—whether Macedonian or Athenian—into a revival of the neglected worship of Dionysus; cf. Campbell, p. 309.

[69] In reference to the words of the messenger describing the revels on Cithaeron, 726ff: "And soon the whole mountain and the wild beasts were in a tumult, and all was in motion through their running hither and thither."

that surrounds us."[70] He likens this religion of the "Bacchanals" to the "added dimension of emotion," the "new reach of freedom" discussed by William James in his *Varieties of Religious Experience*.[71] It is this which makes the play a religious one. In a word it is "faith," which Professor Verrall says is the one thing new in the play, the thing which differentiates it not only from every other drama of Euripides, but from everything else in Greek literature, "the thing, the human phenomenon.... which is, in one word, faith or a faith—religion as we mostly now conceive it, exclusive in belief and universal in claim, enthusiastic, intolerant, and eager to conquer the world."[72] Though the phenomenon is common enough to us, it was apparently unknown to the Greece of the poet's time and was first revealed to him in his last days in Macedon.

It is this, then,—the praise of enthusiasm and inspiration in nature, the personification of exultation in life and emotion in religion—which forms the chief motive of this strange play. The victory of Dionysus over Pentheus, that is, the victory of enthusiasm over reason, the showing up of the defects of human wisdom in comparison with the greater knowledge of the mysterious unknown, all this teaches a lesson no less plain than that disclosed by the victory of Aphrodite in the "Hippolytus" written twenty-three years before. In these two companion plays, two great facts of nature, enthusiasm and love, are personified. These are two great necessities of our human natures, sources of happiness for weary mortals, and they cannot be reasoned away on any rational grounds, nor can they be disregarded without terrible effects, as is exemplified in the fate of both Pentheus and Hippolytus. Euripides

[70] *Op. cit.*, p. 317. His explanation of the play on the theory of Macedonian influence I have followed in the main.

[71] P. 48.

[72] *The Bacchanals of Euripides and Other Essays* (1910), p. 159.

constantly denounced every form of superstition; at the same time he was always opposed to a dogmatic rationalism; and so the "Bacchanals," written at the end of his life, is in a sense the summing up of his position.

Whether the new vision, which seems to have taken such complete hold of the poet, would have been lasting had he enjoyed a longer lease of life, is another question. Even in the play itself are indications of the old iconoclastic spirit reappearing; for example toward the end, in the brief colloquy between Dionysus and Agave,[73] the latter answers with disdain in the line already quoted,

> "'Tis not meet that gods nurse their anger like men."

And that after all the Greek gods are but the contemptible puppets of a vast and indefinable fate is attested by the final verses—which are also appended to several other plays[74]—and which doubtless contain the poet's true sentiments: "Many are the forms of things divine, and many things unhoped for the gods bring to pass. Both what was expected has not been fulfilled and of the unexpected God has found a solution. So hath it happened here."[75] In the "Hippolytus," Phaedra, father and son are all pictured as the puppets of divine caprice; here at the end of the "Bacchanals" Euripides goes a step further and makes not only Pentheus, Agave and the rest puppets of the gods, but the gods puppets of fate.

Thus the play, powerful though it is, contains just such conflicting views as his other works, and so is a true child of the poet. For Euripides, though his dramas were a tremendous factor in carrying on the protest against traditional views of religion which had been inaugurated the preceding century by Xenophanes and Heraclitus, made but little effort to construct a new theology. His mind

[73] 1345ff.
[74] I. e., the "Alcestis," "Medea," "Helena," "Andromache."
[75] 1388ff.

was essentially curious and impressionable to every influence; every thing—nature, society, humanity, religion, philosophy—appealed to him. A recent student of his philosophy has observed that there was scarcely a problem of his day, scarcely a theory in Greek thought before or during his lifetime, of which he did not take account.[76] But though he raised every question he gave a conclusive answer to none, and contented himself with throwing out a crowd of suggestions which at best seem only tentative gropings, and when taken together neither form a consistent whole nor are ruled by any one principle. As Croiset says: "*C'était une intelligence vive et pénétrante plutôt que forte.*"[77] It is quite possible that he had no definite views on religion; he was too great a thinker to yield to the temptation of any one solution, and so like many other great minds he took refuge in mysticism. His nature did not yearn for moral and intellectual anchorage, like that of Sophocles—evidence his shifting, almost kaleidoscopic views of the soul's future: sometimes he simply considers that the problem cannot be solved;[78] again he favors the view of Anaxagoras that is was a dreamless sleep, denying the survival of consciousness;[79] or he paints the usual epic gloomy region of never-ending night;[80] there are passages also in which he asserts that the spirits of the dead still feel with the living,[81] and in others he seems to maintain the Orphic conception, that life is death to the soul and that death is life.[82] This inconsistency in his views impresses us more than any other feature of his mind except his pessimism. As Gomperz puts it: "He delighted to suffer each shifting breath of opinion in turn to seize upon and move his soul."[83] In his defence

[76] Nestle, *op. cit.*, 560. [77] *Op. cit.*, p. 313. [78] "Hippol.," 192ff.
[79] Cf. "Troades," 631ff. [80] Cf. Frag. 536.
[81] Cf. "Electra," 677; "Orestes," 1237.
[82] Cf. Frag. 830 and 639. Cf. on the subject of his eschatological ideas, Adam, *op. cit.*, pp. 306ff.
[83] *Op. cit.*, II, p. 13.

we must remember his life was cast in a period of changing and conflicting thought; the old order of things was passing, but the new was not yet firmly established. It was his destiny to stimulate, to interest, rather than to actually instruct; for his mind was not vigorous enough to embody a system of principles and to cling to them. He was a thinker but hardly a philosopher; and first and last he was a poet, and so in accordance with the Greek idea a teacher also. For Plato says in a beautiful passage that the poets "are to us in a manner the fathers and authors of wisdom."[84] And Aristophanes had already expressed a similar thought when he said that the poet "should conceal what is evil and neither bring it forward nor teach it. For just as children have teachers to direct them, so poets are teachers for grown people."[85] So the religious views of a Pindar, an Æschylus or Euripides, influenced the people deeply. In the "Bacchanals" there seems to be no trace of the great problem which constantly perplexed Euripides—the reconciliation of an imperfectly ruled world with the idea of a benevolent God. But its absence in this play is no guarantee that he had finally found its solution; more probably he never found any light to bring into harmony his intellectual doubts and his moral yearnings. Doubtless much of the pessimism which is evident in many of his plays—a pessimism which at times is synonymous with hopeless despair—is to be explained by this lack of unity.[86]

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[84] "Lysis," 214. [85] "Frogs," 1053ff.

[86] E. g., in the "Hercules Furens," "Hecuba," "Troades," "Andromache" and especially in frag. 452. Adam, p. 311, argues that this pessimism is not entirely due to the political and social changes of the poet's day, for Sophocles, his contemporary, was not affected by it; Gomperz, II, p. 10, ascribes it to the growth of reflection as well as to the unrest of the transition age in which Euripides lived.

THE FATHER OF MONISM.

IT will perhaps interest readers of *The Monist* to have before them the following attempt at an English version of the poem—or rather of the principal fragments of it that survive—in which the father[1] of monism embodied the passion, one might almost say the fury, of his conviction that What Is is One. The verses of Parmenides "On the Nature of Things" are remarkable for two reasons: they are the first thorough-going attempt to prove that reality is a unity, and they are the earliest expression of an idea which was to dominate philosophy with tremendous consequences for nearly two thousand years afterward. The conclusions of the Eleatic school as to the nature of reality were too fantastic to be widely accepted; but the theory stated by Parmenides, the first Eleatic, and never since more vigorously stated, that there is only one way of obtaining scientific knowledge about the world, established itself almost without question. That truths having the certainty of demonstration can only be reached by *a priori* reasoning and never by observation of phenomena, which therefore cannot be the objects of science,—this theory, once promulgated by Parmenides, was taken up into the main stream of Greek thought as a fundamental assumption. Plato and Aristotle shared it, and its validity, supported by the great fabric of the Aristotelian logic, was never seriously attacked until Galileo looked through his

[1] Parmenides was the father of monism rather than the first of monists. Xenophanes was "the first of those who went in for monizing" (Aristotle, *Met.*, A. 5. 986*b* 21), but he was primarily a poet and preacher and had little influence on systematic philosophy.

"optick glass." Even then it survived in part; for it is still true that we have no absolutely certain knowledge except such as can be deduced from general principles. But when science began to advance independently of Aristotle, the domain of the *a priori* was curtailed. We no longer think that no knowledge except the absolutely certain deserves to be called scientific; in investigating the laws of nature we are content with a high and ever-increasing degree of probability. The main interest of Parmenides's poem is that in it a tendency which was to defer that consummation for many centuries first becomes articulate.

It has the strangeness of all origins. No literary document of equal importance bristles with problems apparently so hopeless of solution. There are, to begin with, several difficulties connected with its structure. It has two parts: an exposition (with a proem) of the Way of Truth, and an exposition of the Way of Opinion, of which the first is preserved almost in its entirety, while perhaps one-tenth of the second survives.[2] In the opening lines the philosopher is whirled away in the chariot of the Sun to the abode of a Goddess, who expounds to him two doctrines, a true and a false, "the unshaken heart of well-rounded truth" and "the opinions of mortals." What is the nature of the journey, and who is the Goddess? Why, after she has declared the truth about the universe, should an account which is emphatically stated to be false then be put into her mouth? No certain answer seems possible to these questions. As to the journey, it looks at first sight as if Parmenides were conveyed in the chariot upwards from darkness to the "Gateway of the Paths of Night and Day," and that then he passes through the gateway into a realm of light where the Goddess makes her revelations. But it is just possible to interpret the text as a descent

[2] Hermann Diels, *Parmenides Lehrgedicht*, p. 26. Berlin, 1897.

to the nether world. On this view, which is that of Otto Gilbert,[3] the gate described so minutely in the proem (13-24)[4] is the door of Hades. We must conceive the philosopher as accompanying the Sun on its nightly journey to the under-world, and the Maidens who guide the car as persuading the Goddess to open the gate of Night and Day, which she guards, that they may pass through and the Sun resume his daily course. They then drive on and upwards, leaving Parmenides alone with the Goddess, who is no other than that "Justice" or "Necessity" mentioned in other parts of the poem. There is much to recommend this view, which is interesting as making Parmenides one of the illustrious company of poets, headed by Homer, Virgil and Dante, who have descended to the under-world; but the arguments for and against it cannot be discussed here. Whether the journey be heavenward or hellward, the identification of the Goddess with Justice and Necessity, and again with her who, in the cosmological part of the poem, is in the center of the "rings," "steers all things," and is the creator of the gods (187-192), has great plausibility. Mr. Cornford[5] has ingeniously connected her with the principle on which, in primitive religious systems, the universe is marked out by tabus. But on these points there is likely for some time to come to be more speculation than agreement among scholars.

As to the Way of Opinion, which forms the last part of the poem, and which seems to have contained a system of the world in which concentric spheres or rings of light and darkness played a part, and an account of the birth and decay of gods, of material objects, of animals and of the bodies and souls of men, the difficulty is to explain why Parmenides stated it in such detail. The few frag-

[3] "Der *δαίμων* des Parmenides," *Archiv für Geschichte der Philosophie*, Vol. XX, p. 25. Berlin, 1907.

[4] The figures in brackets in the text refer to the lines of my translation.

[5] Francis Cornford, *From Religion to Philosophy*, p. 217. London, 1912.

ments of it that we possess are not continuous, and I have therefore not translated them all. It appears from them that the cosmology of the Way of Opinion had an affinity to that of the Pythagoreans, and Professor Burnet has sought in this fact an explanation of our difficulty. Parmenides had been a Pythagorean, and was now, he suggests, founding a dissident school. It was therefore "necessary for him to instruct his disciples in the system they might be called on to oppose."[6] If we adopt this view, we may outline the trend of the argument as follows, disengaging it from the archaic language in which it is expressed.

Nothing can have any reality except What Is: for every thought must have an object—thought and its object in fact form an indivisible unity—and the object of a thought cannot be nothing (49, 61-64, 129-131). Further, reality must be eternal, i. e., without beginning and without end. It cannot come into being, because it cannot be produced by nothing, and nothing existed before the existence of that which is real (86-98). Again, no reason can be given why, if it began to be, it should begin at one time rather than another (92-94). And similarly it cannot come to an end (106-108). Thus it is a mistake to attribute any reality at all to the processes of change, growth and decay that we see going on round us (109-112, 135-140). And reality is absolutely single, simple and continuous. It cannot have parts, because, if there were parts, there would be empty gaps between them and thus more reality in some places than in others, which is absurd (113-117, 145-150) We must therefore conceive the substance of the universe as shaped like a sphere (since the spherical is the most unbroken and perfect of forms) with no vacuum anywhere, perfectly stable, with no differences, changes or motions (141-144). This sphere, though its

[6] J. Burnet, *Early Greek Philosophy*, p. 211. London, 1908.

existence is temporally endless, is limited in space; for if it were infinite there would always be "something lacking" to complete the sum-total of reality; but this is impossible (125-128). Thus the prejudices of common sense, which sees differences everywhere—differences of distance, for instance (49)—and thinks that things become and perish and that there is such a thing as change of sensible qualities (135-140), are all false, in spite of the difficulty we have in shaking them off, confirmed as they seem to be by constant experience (42). And not only so, the more refined views of philosophers are false too, particularly the views of Heraclitus of Ephesus, who holds that the only scientific truth attainable is that based on the endless flow of shifting sense-experience (69-76).[7] There is only one way of attaining truth, namely by following reason (45-48). There is, however, one account of the universe, that given by the Pythagoreans, which is so inherently plausible that it must be expounded at length; you must be versed in its details to be able to refute them (167, 168). It is based on a dualism—that of the "light" or "fiery" and "dark" or "heavy" elements — which of course cannot for a moment be accepted, as our argument proves conclusively that all things are One.

But when we have done the best we can with the journey and the Goddess and the Way of Opinion a stumbling-block still remains. It is not only modern readers to whom it seems strange that Parmenides wrote in verse; the fact disconcerted antiquity as well. It was felt that he was essentially prosaic. Why then did he drape his theory in the rich, stiff, hieratic dress of the hexameter, as the old sculptors clothed their idea of deity in stiffly falling lines of stone and bronze and wood? Why did this first

[7] It is generally agreed that the vituperation of these lines is directed against Heraclitus. For the opposition between Parmenides as the philosopher of pure reason and Heraclitus as the philosopher of experience see Emanuel Loew, "Parmenides und Herakleitos im Wechselkampfe," *Archiv für Geschichte der Philosophie*, Vol. XXIV, p. 343. Berlin, 1911.

founder of rationalism begin the custom, which has since had so long and so curious a history, of mixing argument with poetry? The essence of his gospel is "Cleave to the dry light of the intellect, whatever the richness of the facts that strike the senses," and one would think that, than such a gospel, nothing was less suitable to poetry, for which, besides, he evidently had but a meager gift. His technique is clumsy, his images artificial and insipid; his lines jolt and hobble, and he has no warmth of imagination, no glowing colors with which to enrich and soften the bald severity of his subject.

Perhaps it was partly out of opposition to Heraclitus, with his talent for hitting out a striking phrase in prose, that Parmenides chose verse. "Let this gross believer in the trustworthiness of sense-perception string his pedestrian sentences together; my doctrine of the perfect stability, the unbroken unity, of the real demands a form as stable and rounded as itself." Such may have been his feeling. And perhaps the influence of Hesiod went for something. Hesiod had written his account of "Works and Days," of the birth of gods and the ordering of the world, in hexameters, and there is more than one Hesiodic trait in Parmenides. Another influence may have been the Orphic poems current in the Pythagorean school from which Parmenides sprang. But whatever his motives, and whatever his defects as a poet (they have been exaggerated by some critics, minimized by others[8]), our verdict must on the whole be that he was justified. It is not so much that the introduction contains what Diels[9] calls "a powerful conception." That is a matter of opinion; many readers will find it unimpressive where it is vague (and it is nearly all vague), and pointless where it is precise, as in the

[8] Exaggerated by Proclus (*In Plat. Parm.*, I, 665, Paris, 1864), by Plutarch (*De Rat. Aud.*, 3, 45B, *Quomodo Adul.*, 2, 16C), by Philo (*De Prov.*, II, 39), by Cicero (*Ac.*, II, 74); minimized by Bergk, *Kleine Schriften*, II, 10). See Diels, *op. cit.*, pp. 4ff.

[9] *Ibid.*, p. 7.

description of the gate. Parmenides's real justification is the intensity of his passion for the truth.

All philosophers, no doubt, are impassioned for the truth, but not all philosophers are possessed of a passion for the compulsive force of argument. When a man has hit upon an abstract argument in which he can see no flaw and which leads to conclusions violently opposed both to common sense and to the views of other philosophers, his zeal is apt to take an almost religious tinge. Up to a point the love of reason seems, indeed, to be implanted in the human breast. The most irrational of men, those most impatient of logic, take an unconscious pleasure in the struggle to elicit conclusions from premises; the motions of their minds, sluggish though they may be, are always fumbling after some rudiments of a chain of inference. But let the beauties of logical connection once become the object of conscious admiration and be deliberately pursued for their own sake, and there are men who, having tasted blood, will stop nowhere. In them our natural, unconscious pleasure in ratiocination is heightened to the *n*th power; they exalt the value of consistency above everything in the world. It becomes a fixed idea; they give up everything for it; they embrace an abstraction with the abandonment with which the lover embraces his mistress, the devotee his god. The world, to them, is well lost for logic. Are the facts against them? So much the worse for the facts, they cry. They are the martyrs of reason; they are sublime. And they are more than sublime, they are right; for the progress of humanity depends in the long run on the love of reason.

Parmenides was such a man, and his verses are poetry because they are moulded by this passion. That is why they are least good in the half-mythological, half-allegorical preface with which, for reasons at which we can but dimly guess, he leads up to his belief that nothing Is

except What Is, and best when he is in the thick of his argument, stumbling, stammering, repeating himself, and wrestling with the reluctance of the language of his day to express his ideas. In his desperate anxiety to make his point clear his verses become rough and harsh, and it is then that they take on a certain sublimity, as of igneous rocks compressed and thrown up by tremendous subterranean forces.

It will perhaps be objected that he could have made his point clear more easily in prose. To this an answer has been provided by a very different poet, Alexander Pope, who explains, in the introduction to the "Essay on Man," that he found he could actually express his philosophical ideas more concisely in verse than in prose. For the labor of throwing a theory into verse has at any rate this merit, that the philosopher who is diffuse is lost. The nature of the medium compels him to grind and sharpen his thoughts until, purged of all superfluities, they attain the utmost sparenesss and compactness of which they are capable. So true is this that far from blaming Parmenides we should wish that modern philosophers would imitate him and Lucretius and compose in verse; their arguments, if like Lucretius and Parmenides it is their arguments they are in earnest about, might be improved by the discipline. On the other hand if like Pope they care not a pin for the argument but greatly for the opportunities of verbal decoration, conceivably some entertaining poetry might be produced.

Passion, then, and conciseness—passion in spite of the lack of imaginative heat, conciseness in spite of clumsiness and repetition — are Parmenides's most striking qualities. That the quatrains into which I have transposed him preserve more than the dimmest reflection of these qualities it would be too much to hope. Hardly can the color and life of a phrase be conveyed from one living

language to another, much less from an ancient to a living language. The only respect in which the translator can hope to be a faithful mirror is in giving, feature by feature, the connections of his author's thought; and, since in doing this he may be allowed to take the necessary liberties with his text, I have had no compunction in condensing here, amplifying there, and occasionally omitting a line or two altogether. Without trying to be always literal I have aimed at omitting no point of importance. The real difficulty was to find a vocabulary not too remote in spirit from the original. In the case of an early philosopher this difficulty is especially acute.

Parmenides was 65 years old when he came to Athens and talked with Socrates who was then a youth of 18 to 20 —a fact[10] which gives us 516-514, B. C., as the date of his birth—and at the time he taught the process of stretching the words and phrases of ordinary speech to fit philosophical ideas had scarcely begun. In the absence of a technical vocabulary thought both outruns language and is crippled by it, so that our more abstract colorless words which have a long philosophical evolution behind them seldom quite fit the early thinker's meaning. It is not that his ideas are vaguer than ours, but their vaguenss is of a different kind. Ours is a washed-out vagueness, theirs a dense, packed vagueness, pregnant with the germs of future growth. Thus the translator is in a dilemma. He cannot, since the thoughts he is to render are philosophical, altogether avoid words which, like "reason" or "infinite," have done philosophical duty for centuries; yet he knows that such words distort the spirit of the original, because their fifth-century Greek equivalents are only just beginning to have a specialized philosophical color. For this reason I have employed such words as sparingly as pos-

[10] Plato is the authority for this (*Parm.*, 127b). Diogenes's statement that Parmenides "flourished" 504-500 B. C. does not seem a sufficient ground for questioning Plato's accuracy.

sible. But then another danger arises. Modern technical phrases may strike a false note, but if we do not use them we risk blurring the outlines of the technical questions the author is struggling to state. Above all in Parmenides's poem the student is fascinated by the spectacle of later ideas stirring in embryo,—as where he announces in one place (62): "It must needs be that what can be thought and spoken of is," in another (129) that thought and "the goal of thought" (i. e., that for the sake of which the thought is, that to which it is directed, its *object,* as we say) are one and the same, and again (135, 158) that certain things, e. g., becoming and perishing, are mere names. Here we seem to catch logic and epistemology almost in the act of being born. What is meaning? What are propositions? Must not the object of every judgment be something real? Can an object of consciousness be conceived apart from a conscious subject? These vast questions are enfolded in the verses of Parmenides as the oak in the acorn. Another instance is the argument, on which he bases the oneness of What Is, that there cannot be more reality in one place than in another (51, 52, 113-116, 145-152). We may trace here the germ of Zeno's antinomy of the great and little,[11] which in its turn is the germ of that supposed self-contradictoriness of the infinite divisibility of space which has played so important a part in modern philosophy. Such problems hovered before the mind of Parmenides as in a glass darkly, and I shall be content if in my translation some faint image of them can still be discerned.

PARMENIDES ON THE NATURE OF THINGS.

I. The Journey.

And so, behind that team of sapient steeds,
On the illustrious road divine that leads
The wise world-wanderer to his heart's desire,
The straining car that bears me onward speeds;

[11] Simplicius, *Ph.*, 141, 1, quoted by Diels, *op. cit.*, p. 83.

On, ever on, its forerunners a band
Of Maids. The axle-tree even as a brand
Smouldered, and shrilled a music as of pipes
To the twin wheels that rac'd on either hand,

When, once again from the dim house of night
Hastening upwards to the realms of light,
The Daughters of the Sun-God cast away
Their sable kerchiefs and their heads undight.

Here stands the portal where the paths divide
Of night and day; stony the threshold, wide
The lintel; filled with mighty doors it is,
By which the great Avenger doth abide,—

Justice, who grasps the ever-changing key.
Her did the Maids pursuade with honey'd plea
To slip the pin and smite the bolt away;
And the doors sprang asunder instantly,

As one by one the posts of knotted brass
Back on the hinges rolled their thick-wrought mass,
Yawning to let the Maids and steeds and car
On through the gulph and up the highway pass.

And me the Goddess greeted, and with look
Benign my right hand in her right hand took,
And "Hail!" she said "O Youth, for thou art sped
Divinely hither; hail!" and thus bespoke:

" 'Tis no ill doom hath led thee on this road
Far from the common track of mortals trod,
But, tended by those deathless Charioteers,
Justice and Right bring thee to my abode.

"Thine be it now the steadfast heart to know
Of Truth well-rounded, and the ebb and flow
Of Semblance in men's minds; no sure belief
Is in it; thou must learn it even so,

"That, testing all things, so thou may'st declare
The Things that Seem, how men should judge they Are;
Yet must thou set a curb upon thy thought,
And ever of illusion's path beware,

"Lest, poring all too closely on that maze,
The force of use and wont distract thy gaze,
And, droning in thine ear an idle din,
Hurry thee babbling down deceitful ways.

"But hold to Reason when dispute is rife,
And thou shalt know there is not any strife
Can shake this much-tried argument of mine:
This is the proof, this is the Way of Life."

II. The Way of Truth.

"See how thought makes the far thing near! 'Tis plain
Thou canst not cleave the All that Is in twain;
'Tis not a thing of parcels that may be
Scattered abroad, nor yet heaped up again.

"Come, ponder deeply these two Ways of Thought
By which alone all knowledge must be sought,—
The Way of Truth and Suasion hand in hand,—
That What Is Is and Not to Be is Naught,—

"And then the Way of those who take for true
What neither tongue can tell nor thought pursue,—

That something Is Not and must needs Not Be:
That path as wholly blind thou shalt eschew.

"For how can what Is Not be ever known,
Since to be thought on and to be are one?
For everything may be, nay needs must be,
Which speech can name or the mind think upon;

"But what Is Not in Being hath no part.
Lay deeply, then, these precepts to thy heart,
That from the snare of false opinion's Way
Thy firm-set feet may evermore depart;

"Much more from that which witless mortals stray,
Double-faced fools who know not what they say,
But sightless, shiftless, lacking pilotage,
Palsied and deaf, they hither and thither sway;

"Dull herds, to whom the Thing that Is doth seem
The same as what Is Not; and then they deem
The same is not the same, and all the world
Whelm in an ever backward-flowing stream.

"Of this be sure, there is no argument
Shall prove What Is Not Is; be thou content
To curb thy wit from searching out that way,
On the one Way of Being wholly bent:

"Whereon is set full many a sign to tell
That All that Is is indestructible,
Nor ever was created; for complete,
And endless is it, and immovable.

"It never was nor will be; it Is now,
Whole, one, continuous. Its sources how

Wilt thou search out? Or whence draw its increase?
'From that which was before it,' sayest thou?

"But Nothing was before. From Nothing, then?
But this may not be utterèd of men,
Nay, nor conceived, that Nothing ever was.
And if from Nothing, what should choose the when,

"What fix the soon or late, by what decree,
When that which Is should start to grow and be?
Wherefore hold fast this Truth: the Thing which Is
Or all in all or not at all must be.

"Nor will the force of true belief allow
Out of what Is Not aught save Naught to grow;
Therefore things neither perish nor become;
Justice hath fettered them nor lets them go.

"Is it, or Is it Not? All must abide
That test. Let stern Necessity decide,
Who saith 'It Is' is true, and casts 'Is Not'
As nameless and unthinkable aside.

"How then could That which Is ever arise?
For if it was, it Is Not; and likewise
It can not be some day about to be;
Who saith 'It will be' that It Is denies.

"And thus becoming, like a flickering flame,
Is quite extinguished, and that other name,
Destruction, is an empty sound; What Is
Hath nor an end nor source from which it came.

"And how divide it where no difference is,
Nor more of it in that place than in this

To hold its unity apart? Thus all
Is full of it. What Is cleaves to What Is.

"And therefore, as in mighty bonds comprest,
Without beginning, in an endless rest
Is that Which Is, since we have spurned afar
Birth and destruction at the truth's behest.

"Ever the same and ever in one stay
(For strong Necessity hath every way
Fastened the limit round It) It abides,
And changes not, wrapt in Itself alway.

"And straitly bound in limits, as is fit,
The All that Is can not be infinite;
Else It would lack all things; but, lo, It lacks
Nothing; Naught can be added unto It.

"Thought and the goal of thought, these two are one.
For never shalt thou find beneath the sun
A thought exprest without the Thing that Is;
Since no things are, nor shall be, no not one,

"Save those which into the one perfect round
Of moveless being fate hath strictly bound.
Wherefore those names that mortals in their speech
Fix, and believe them true, are empty sound,

"Telling of birth and of destruction,
Of how things change their places and are gone,
How now they are, and now forsooth are not,
And how fair colors fade that brightly shone.

"But since What Is hath an extremest bound,
'Tis like a massy sphere's unbroken round,

Which, from the center poisèd equally,
Complete and equal every way is found.

"There is no Nothing anywhere to break
Its even unity, or greater make
Its plenitude in this place than in that;
It can not here be strong and there be weak;

"For not in any wise can That which Is
Be present more in that place than in this;
Out from the center to the utmost verge
All equal is and all inviolate is."

III. The Way of Opinion.

"Thus far the Truth with reasons sure and clear
Have I declar'd, and next what Shows appear
To mortal men must be in order told:
Do thou to my deceitful song give ear.

"Two Forms there are that mortals have in mind
To name, and naming one they wander blind;
They part the twain as opposite in shape,
And to each opposite are marks assigned.

"To one they give the Heaven's ethereal flame,
Gentle, exceeding light, ever the same,
Itself like to itself; contrariwise
To another Form they give another name,—

"The heavy body of darkness, solid night,
Set over against the influence of light.
(I tell thee all as all most likely seems,
That no man's subtlety may pass thee quite.)

"And then, their names being given to night and light
And to whate'er belongs to either's might,
Since neither in the other hath a share
All things are filled with equal light and night.

* * *

"The substance of the Heavens shalt thou know,
And all the high fixed signs that in them glow,
And those effulgent labors of the Sun—
Whence come his cleansing fires and whither go;

"The wandering Moon too, with her pale round face,
Her works and substance shall thy cunning trace;
And how the Heavens were born, by what dread law
They bind the world and hold the stars in place.

"And thou shalt know how Sun and Moon and Earth
And uttermost Olympus sprang to birth,
And all-embracing Ether, and the might
Of burning Stars, and the Heaven's milky girth.

* * *

"With unmixed fire are fill'd the inmost rings;
The next with darkness; and the appointed springs
Of flame gush in between; and in the midst
The Goddess is who sways and steers all Things,

"Urging all creatures on the sweets to prove
Of mating and the painful fruits thereof,
Male unto female, female unto male;
For of all Gods the first she fashioned Love."

SYDNEY WATERLOW.

LONDON, ENGLAND.

CRITICISMS AND DISCUSSIONS.

"CHRISTIANITY OLD AND NEW."

This is the title of a course of lectures delivered by Prof. B. W. Bacon, D.D., of Yale Theological Seminary, at the University of California. In order to appreciate and understand the point of view of the lecturer it is necessary to recall the following facts: In the *Hibbert Journal* for January 1910 the present writer had an article entitled "The Collapse of Liberal Christianity"; and a year later another article on the same general theme under the caption, "Whitherward? A Question for the Higher Criticism," carrying the argument a little further. The purpose of both articles was to show that liberal Christianity had failed in its attempt to find a historical Jesus. The main proposition was that liberal Christianity began its course by repudiating the Christ of the church and by planting itself on a purely human Jesus who, of course, it took for granted was a historical person. It was pointed out that it had been engaged for over a hundred years in seeking for this historical Jesus, because he was necessary to the existence of the movement as a protest against orthodox Christianity. The writer admitted at the time and all along that the title of the article was ambiguous and was therefore liable to be misunderstood. But the meaning attached to the words was fully explained in the course of the article, which was not to assert that liberal Christianity or liberal thought had collapsed as a whole, but only that the attempt to find a historical Jesus had failed.

In the *Hibbert Journal* for July 1911 appeared an article by Professor Bacon, entitled "The Mythical Collapse of Historical Christianity," in which there was a misunderstanding of the above. A meaning was attached to the "collapse" never intended by the writer and a consequent wrong impression given to the readers

of the Journal and the public generally. Professor Bacon called upon "those who were reading in a contrary sense such momentous signs of the times as the modernist movement, extension of the voluntary principle in church support, church federation, and the new impetus in religious education, not to be suddenly dismayed." That is to say, Professor Bacon represented the writer as asserting that modern thought had suddenly come to a standstill! What was meant by the title was one thing and one thing only, that the liberal search for a historical Jesus had proved a failure. In other respects the writer believed with Professor Bacon that liberal Christianity "so far from being in danger of collapse is advancing to-day by great strides towards the place of leadership and authority in modern religious life." And not only so, but he regards the effort of Professor Bacon and others to stop with a historical Jesus as a failure of liberal Christianity to be true to its principles. Liberal Christianity in the large sense of that phrase is, as Professor Bacon says, "but beginning its career, and rejoices as a strong man to run a race." And in pursuance of the course under the leadership of the Dutch school of criticism, of which Professor Bacon makes no mention either in his *Hibbert* articles or in this book, it is fulfilling its mission and carrying out its principles to their legitimate conclusion. Why should it come to a standstill with the historicists of Germany? Surely the doctrine that the central figure of the New Testament was a historical person is not a finality. The writer believes that those who refuse to stay with the historicists and who go on to interpret the New Testament in a more spiritual sense than is possible on that theory are *par excellence* liberal Christians. They are just now enduring what all must endure who venture to call in question the results of "established scholarship," and to affirm the symbolic character of the Gospel story.

In his new book, *Christianity Old and New,* Professor Bacon shows that he has come to see the mistake he made as regards the word "liberal." Not that he acknowledges it in words—that perhaps would be too much to expect—but it is implied in what he says. He contrasts the view the writer advocated in his articles with that of President Eliot of Harvard University in his essay entitled "The Religion of the Future." President Eliot has for a generation and more been the most distinguished Unitarian in America. He has been looked upon as the leader of Unitarians all that time, and his prognostication of what in his judgment Christianity is to be in the future has been universally accepted by Uni-

tarians all over the world as embodying for them the truth. It is not necessary to say that Jesus is presented by President Eliot simply as a man, and that the "religion of the future" which he puts before the world is a Christianity denuded of all those elements which have made it the Christianity of the church. Not only does it lack the virgin birth and physical resurrection, but also those doctrines of incarnation and atonement which have ever been regarded as vital to Christianity. Professor Bacon's summing up of President Eliot's position as set forth in an article in the *Harvard Theological Review* (October, 1909) is worth quoting (p. 38): "President Eliot's reconstruction presents the distinctive type of what has claimed for itself, and has sometimes been accorded, the honorable name of 'liberal' Christianity. To him the mystical doctrines of personal religion, the doctrines of incarnation, atonement, immortality, represent mainly 'pagan' accretion. To restore to Christianity its true message for our times we must trace it back (thinks President Eliot) to its 'Hebrew purity' in the ethical teachings of Jesus. As for what are termed 'the consolations of religion' they will be mainly found in '....a universal good will, under the influence of which men will do their duty, and at the same time promote their own happiness. The devotees of a religion of service will always be asking what they can contribute to the common good....The work of the world must be done, and the great question is, shall it be done happily or unhappily?' Much of it is to-day done unhappily. The new religion will contribute powerfully toward the reduction of this mass of unnecessary misery, and will do so chiefly by promoting good will among men."

This, then, is "liberal Chritianity," at least it is the "liberal Christianity" which the writer had in view and which plants itself on a Jesus who was purely human. Such a Jesus is a necessity for it. Hence the efforts which the Liberals have been making for the last hundred years and over to discover a historical Jesus. Three things are worthy of attention at this point. The first is that this Christianity which President Eliot predicts is to be the "religion of the future" is a distinct break with the Christianity of the church of the past and of the church of the present. In all ages Christianity has been regarded as the religion of redemption; but redemption is eliminated from this conception of it. Professor Bacon recognizes this when he says that it "goes more than half way to meet the Reformed Synagogue and the liberal Ethical Society." None of the great theologians of the past would have recognized this as

Christianity. Professor Bacon asks, "Can this really be the 'Gospel' in which there is not one word of what Paul describes as 'the ministry of reconciliation, committed unto us' as ambassadors for God, 'how that in Christ God was reconciling the world unto himself, not imputing unto men their trespasses?'" And if Paul would have failed to see in this "religion of the future" his Gospel, so we may be sure would the great fathers and theologians of the church have failed. To one and all Christianity was a redemptive scheme, and they would not have understood a religion which had no incarnation or atonement, and which presented itself to men merely as a system of ethics.

It is safe to say that no church of the present would accept President Eliot's summation of Christianity. Neither the Roman Catholic nor the Greek church could do so. The Anglican church in all lands, the various branches of the Presbyterian church throughout the world, the numerous Methodist bodies, all with one accord would have to repudiate this version of Christianity as not Christianity at all. They would doubtless, as does Professor Bacon, recognize "the nobility of the ethical ideal" involved in it; they would say with him that it is "good as far as it goes, but it does not touch bottom"—and that because it does not meet man's deepest need which is for redemption rather than for ethics. Professor Bacon admits that "it may rightly claim to reflect in large measure the teaching of Jesus"; but it has always been held that the teaching of Jesus so called does not contain the distinctive Christian Gospel, because that consists of something which Jesus *did* rather than what he is reputed to have *said*. And hence the teachers of the church have gone to the Epistles of Paul instead of to the Sermon on the Mount for Christianity's distinctive message.

Some words of Professor Royce, a distinguished member of the faculty of Harvard University of which President Eliot was the head, are worthy of attention in elucidation of this point. In an essay on "What is Vital in Christianity" he tells us: "What is vital in Christianity depends upon regarding the mission and the life of Christ as an organic part of a divine plan for the redemption and salvation of man. While the doctrine of Christ, as his sayings record this doctrine, is indeed an essential part of this mission, one cannot rightly understand, above all apply, the teachings of Christ, one cannot live out the Christian interpretation of life, unless one learns first to view the person of Christ in its true relation to God, and the work of Christ as an entirely unique revelation and ex-

pression of God's will. The work of Christ culminated in his death. Hence, as the historical church has always maintained, it is the cross of Christ that is the symbol of whatever is most vital about Christianity."

Nowhere, perhaps, is the issue involved in this matter so clearly set forth as in the following words of Professor Royce in this same essay. "The question is simply this: Is the Gospel which Christ preached, that is, the teaching recorded in the authentic sayings and parables, intelligible, acceptable, vital, in case you take it by itself? Or does Christianity lose its vitality in case you cannot give a true sense to those doctrines of the incarnation and the atonement which the traditional Christian world so long held and so deeply loved? And furthermore, can you, in the light of modern insight, give any longer a reasonable sense to the traditional doctrines of the atonement and the incarnation? In other words: Is Christianity essentially a religion of redemption in the sense in which tradition defined redemption? Or is Christianity simply that religion of the love of God and the love of man which the sayings and the parables so richly illustrate?" Professor Royce maintains that "the whole authority, such as it is, of the needs and religious experience of the church of Christian history stands on the traditional view that the essence of Christianity consists first, in the doctrine of the superhuman person and the redemptive work of Christ, and secondly in the interpretive life that rests upon this doctrine. The church early found, or at least felt, that it could not live at all without thus interpreting the person and work of Christ."

The second point that needs to be carefully noticed is that Professor Bacon does not accept the historicity of the alleged events on which the doctrines of incarnation and atonement have rested in the past. This comes out both in his *Hibbert* article and in the book under review. New Testament criticism under his able hands gives us "Jesus as teacher and leader of humanity toward the ideal of the brotherhood of the race under the fatherhood of God."[1] On the preceding page he implies that the historical criticism of the New Testament of which he is such an ornament reveals Jesus as "truly human." He claims to be as free as any one to discard legendary elements in Gospel story, such as the virgin birth or physical resurrection. He declares that he has no "prejudice whatever against recognition of the mythological element in the New Testament." He sets all these aside just as much as does President

[1] *Hibbert Journal*, Vol. IX, No. 4, page 75.

Eliot, but he does not follow President Eliot in reducing Christianity to a mere ethical system. It is to be presumed that he would agree with Professor Royce in regarding the "reduction of what is vital in Christianity to the so-called pure Gospel of Christ, as he preached it and as it is recorded in the body of the presumably authentic sayings and parables as profoundly unsatisfactory"; but he no more accepts the historicity of those alleged facts on which the redemptive element in Christianity has all along rested than does President Eliot. The redemptive element comes from the "transcendentalized Messianism of Peter and the Hellenistic incarnation doctrine of Paul." These are the moulds, so to speak, into which the preachers of the new religion put their thought of Jesus. To use Professor Bacon's own words, "We cannot conceive any other vehicle of thought or speech than myth through which the preachers of the new religion could give utterance to their undisciplined sense of the teleological significance of what they themselves had witnessed." In his *Making of the New Testament* (page 52) he says that the mystery-religions of paganism formed "the only mould for Christology." That is to say, there was no basis in history itself for the Christology of the New Testament and the church, that basis is found in the apostolic interpretation of Peter and Paul which makes these apostles the real founders of Christianity and not Jesus. This agrees with what Professor Bacon says in the opening of his *Story of St. Paul*: "Christianity as we know it is St. Paul's Christianity." Now this comes as near as can be to the doctrine which Professor Bacon criticised in his *Mythical Collapse of Historical Christianity*. The Messianism of Peter and the Hellenistic incarnation of Paul were subjective experiences of these two apostles. This is divided by the thinnest of thin lines from the doctrine in the two first articles, and contains really all the writer contended for. What has been looked upon as Christianity from the beginning until now does not rest upon facts of history, but upon subjective experiences. Jesus was a man only, but Peter and Paul interpreted him as God. His influence upon them was so great that they could express what they felt about him only by taking advantage of what the devotees of the mystery-religions said of Adonis and Attis and Osiris. This he calls "the golden background of dogma, Pauline and later, against which the historical figure of Jesus has been seen projected by those who transmit the portrait to us." It is not the portrait itself. It is the

"apostolic gospel about Jesus, the Petrine and Pauline interpretation of the significance of his person, his experience, his fate."

The difference between him and President Eliot is that the latter "has no use for it [the gospel about Jesus] but to cast it as rubbish to the void." Professor Bacon's gospel is *about* Jesus, not what Jesus was himself. The New Testament critic is more concerned with ideas than with facts. "When he discriminates concrete fact and event from the contemporary interpretation which they received, it is the thought rather than the thing which concerns him.... In this field, it is true, historical facts are not unimportant, because when properly sifted they fail to be classified and interpreted in accordance with modern experience by modern standards. But contemporary judgments of the significance of facts, inferences, convictions, faiths, doctrines, are more important." Professor Bacon thus separates himself from the liberalism which makes its watchword the cry, "Back to Jesus." He follows the pragmatists in that it is the worth of Jesus to the apostles and not Jesus himself which is the important thing. He enters a much-needed protest against the liberal Christian of the President Eliot type who "has but one standard of value, historicity." To throw away the mythical and legendary element which he finds in the New Testament is for him to cast its most precious portion into the sea. He tells us that what would be left would be "an admirably simple summary of human duty," for its "morality would be the law of love," and its "emotion would be of two kinds: First, trust in a Heavenly Father,.... second, loyalty to the historic Jesus as a sublimely consistent and heroic leader of the world into its ideal and ultimate social order." To this reconstructed Christianity he would not deny the name Christian, just because it "places the historic Jesus in a position of permanent supremacy." But it would not be the Christianity of the New Testament or of the church, for that "historically is a gospel *about* Jesus, originating with the resurrection, which was not a historical fact, but a psychological experience of primitive believers under Greek influences."

The cry "Back to Jesus" which has been the watchword of liberal Christianity—"Christianity as Christ preached it"—has "overleaped itself and fallen on the other side." In the most emphatic way Professor Bacon declares that "the Christian religion did not originate with the earthly life of Jesus. That is an idea which arose after the period of the apostles in the age of the Evangelists such as Mark. Our religion began with the manifestation of the

Son of God which was not a physical but a psychical experience. It began with the cross and resurrection, the doctrine about Jesus." There is little profit in contending about names. But from all this it appears that Professor Bacon agrees with the statement that liberal Christianity has collapsed in the sense that it is not Christianity at all, not Christianity as the New Testament presents it, not Christianity as it has been understood in all the ages of its history. It appears, therefore, that Professor Bacon admits all that the writer's two *Hibbert* articles contended for. The difference is unimportant. Both say that the liberal Christianity represented by President Eliot in his "Religion of the Future" has "collapsed." The proposition of the writer is that it has "collapsed" because it has not been able to find a historical Jesus; Professor Bacon's is that the historical Jesus whom he thinks criticism has discovered does not give us Christianity, for that is "the doctrine *about* Jesus, the interpretation given by primitive believers to the work of God effected by the spirit of Jesus. His death, his resurrection, inwardly experienced by these men as 'the power of God unto salvation'—these are the most important data in all the psychology of religion." Much opprobrium has been cast upon Professor Drews for speaking of the "Christ-myth," but Professor Bacon cites with approval a declaration of "a great scholar of our time, describing the redemption doctrine of the Pauline missionary preaching: 'This whole point of view is a myth from beginning to end, and cannot be termed anything else.... It is the story of a God who had descended from heaven.'"

It has been assumed that Professor Drews denied the doctrine of redemption because he spoke of the "Christ-myth"; with how much injustice the following quotation will show: "To think of the world's activity as God's activity; of mankind's development, filled with struggles and sufferings, as the story of a divine struggle and passion; of the world-process as the process of a God, who in each individual creature fights, suffers, conquers and dies, so that he may overcome the limitations of the finite in the religious consciousness of man and anticipate his future triumph over all the sufferings of the world—that is the real Christian doctrine of redemption."[2] Professor Bacon tells us that Paul's interpretation of the career of Jesus was what it was required to be in order "to fit the capacity of a pre-philosophic age." It was not therefore the truth, but a symbolic representation of the truth adapted to minds incapable

[2] *The Christ-Myth*, page 298.

of apprehending the truth in its pure philosophic form, a concession to immaturity. What was the redemption which Paul tried to set forth by using the phraseology of the mystery-religions of paganism? Was it the inward work of God effected through the spirit of Jesus only in the minds of the primitive disciples, or was it the world-process of redemption of which Professor Drews speaks? Suppose that what the disciples experienced was a part of this world-process. Which, then, would be the most comprehensive idea of redemption? The historicity of Jesus cannot be necessary to the experience of redemption, for it is an experience which has been felt all over the world. The Roman poet Ovid, speaking of the god Æsculapius, sings:

> "Hail, great physician of the world! All hail!
> Hail, mighty infant! who in years to come
> Shalt heal the nations and defraud the tomb.
> Thy daring art shall animate the dead,
> And draw the thunder on thy guilty head.
> Then shalt thou die; but from thy dark abode
> Shalt rise victorious, and be twice a god."

The rising of Adonis from the tomb was celebrated in words which have been versified as follows:

> "Trust, ye saints, your God restored,
> Trust ye in your risen Lord;
> For the pains which he endured
> Our salvation have procured."

This is the mould, according to Professor Bacon, into which Paul put his experience of Jesus, but is the experience which Paul and Peter and the rest had the only genuine experience? Suppose we say that both alike are symbolic representations to set forth the cosmic story of the divine struggle and passion, would we not have a grander idea of redemption than that which Professor Bacon stands for? He says in condemnation of President Eliot's repudiation of Peter's Messianism and Paul's doctrine of incarnation that "the very last thing the true critic and historian of religion will do with 'mythical' interpretations of genuine experience is to throw them away." May not the experience described by Ovid and in these lines about Adonis—words which might be sung in any Christian church to-day—express a genuine experience expressed as Paul did his experience in unphilosophic form? No one claims that Æsculapius or Adonis was a historical person, but that does not in-

validate the experience any more than Paul's use of the mystery phraseology invalidated his experience. We are thus led to the crucial question of this whole discussion: Is the historicity of Jesus necessary to the genuineness of Paul's experience?

This is the third question that needs careful attention. I offer the following remarks: First, I find nowhere in this book or in any of Professor Bacon's writings any attempt even to prove the historicity of Jesus. He takes it for granted without an atom of proof. For example, on page 97 of this book he says that "Christianity prevailed because of its more solid basis of historic fact." How does he know that? The assumption is that what succeeds in our world must be based on historic fact. Now there is not a scrap of evidence for this proposition. It is pure assumption. The whole history of the world disproves it. Have we not all learned long ago that the world is led on from stage to stage of progress and knowledge "through illusion to the truth"? Suppose that we grant that Christianity would never have succeeded but for the belief in a historical Jesus, that does not prove that the historicity is a fact. It may, if we understand the times properly, prove the opposite. What if it were another case of concession to immaturity? Could Christianity have conquered the rude tribes that overran the fair fields of the old world, the Huns and Goths and Vandals from the North, had it been presented to them in the form that would satisfy Professor Bacon? Suppose that the story of a historic Jesus arose gradually as a higher type of religion declined, and prevailed just because it was a lower type. The question is, what was the earliest form of the faith? In the middle and latter part of the second century we have a creed setting forth the substance of the faith as a series of alleged historic facts, and we find great churchmen like Tertullian and Irenaeus planting themselves upon that creed and defending it against all comers. This was about the time when the canonical Gospels and Acts were taking shape and slowly gaining authority over the mind of the age. But there were other gospels and acts among the people. When we go back to the second century we do not find one church only, but the growing Catholic or central church in violent conflict with churches or communities which it looked upon as heretical. These heretical communities had their gospels and acts which were circulated among the people or members of these heretical communities. The Catholic or central church triumphed over these communities, and our canonical Gospels and Acts are the marks of the triumph. The

Romanist is right in his claim that the church gave the scriptures to the people in the sense that the canon of scripture was fixed by the church; that is to say, the question whether such and such books were to be admitted to, or excluded from, the canon was determined by the church. We should make a mistake if we came to the conclusion that it was always intrinsic merit that decided the question. To be convinced on this point one has only to read the reasons why there are only four Gospels given by Irenaeus himself in his work against "Heresies."

It is always foolish to find fault with the course of history, and no doubt the triumph of the Catholic or general church and the consequent triumph of the canonical scriptures served some good providential purpose. At the same time it is often the duty of after ages to go back and pick up something of value that has been left behind in the onward march of events. In the Gospels and Acts of these heretical communities we have an instance of this. Besides the four canonical Gospels of Matthew, Mark, Luke and John, there are the apocryphal Gospels according to the Egyptians, according to the Hebrews, according to Peter and others; and besides the canonical Acts there are the Acts of John, the Acts of Thomas, the Acts of Peter, the Acts of Paul, and others; and these are only parts that have come down to us of what was once a large literature.

The older view of the controversy of the second century is that it was over an elaborate attempt to alter the Christian faith as it had been handed down in the church from the beginning; that the Christian faith was from the first a body of historical facts such as we have in the Old Roman Symbol and the Apostles' Creed, just as it was the older view of Gnosticism that it was an outgrowth of Christianity, a heresy which gradually arose to trouble the church. It is just at this point that we are obliged to suspend our judgment, if not to reverse it altogether, and to regard the summarizing of the faith on the part of the general or Catholic church as the innovation upon an older form of the faith. The heretical Gospels and Acts, therefore, would have to be regarded as the oldest product of the Christian movement in the sense that they set forth the original form of the faith, and that the summary of alleged historical facts by Tertullian and Irenaeus, culminating in the Old Roman Symbol and finally in the Apostles' Creed, is a later heresy.

If we read the Old Roman Symbol and the Apostles' Creed in the light of these heretical Gospels and Acts, we shall discover

the meaning of their various clauses, for we are walking over the grave of a buried controversy; every clause is directed against what was regarded as an innovating error, but was in reality the older view. It was the Old Roman Symbol and Apostles' Creed which were innovations; it was the view of the Gnostics represented by Marcion which was the conservative view. It has been the opinion of scholars up to recent times that the Old Roman Symbol "was already in use in Rome when he came there in 140 A. D., and joined the Catholic church";[3] but this scholar shows that all the passages which have been relied upon to bear out such an opinion are seen "not to bear the interpretation put upon them."[4] "There is no evidence in them nor is there evidence anywhere that Marcion knew and accepted the Old Roman Symbol." And one may add there is no evidence that this rule of faith was old and well known. It was a new thing, and was just coming into recognition. It had not existed from the beginning. The same scholar shows that neither Justin Martyr nor any of the other apologists of the day knew anything of the Old Roman Symbol as a rule of faith. There is evidence that the notion that the faith consists of certain historical facts is growing, but there is no evidence that it had reached the stage of a fixed creed such as it very soon became. There is mention in Justin of the birth, crucifixion, death, resurrection and ascension as historical facts. There is mention also of Pontius Pilate under whom the crucifixion took place. That is to say, the idea that Christianity consisted of a series of historical facts had evidently become fixed in the mind of the church by the time of Justin Martyr, but when we go back to Ignatius in the first quarter of the second century, while we find him setting forth a strictly historical interpretation of the Gospel, at the same time he does it as though he was stating something new, not what was old and well established. He "protests too much" for one who stands for the old truth. The Old Roman Symbol and the Apostles' Creed were evidently an expansion of an early baptismal formula which was simply "Into the name of the Lord Jesus," or "Into Jesus Christ." This was sufficient for the Jews as their God was the God of the Christians as well, which is evidence that the name "Jesus" was a name for a divine being, and not of a human historical person. When the heathen or Gentiles became Christians it was necessary to add the name "God," as their God was not the God of the Chris-

[3] McGiffert's *Apostles' Creed*, page 58.
[4] *Ibid.*, page 68.

tians, and the word "spirit" would naturally follow. The baptismal formula would thus be, "Into God and Jesus Christ and Holy Spirit."

If there is one thing that has been proved by recent scholarship it is that the Gnostics were the first Christians. The various clauses of the Old Roman Symbol and of the Apostles' Creed were added as the idea of an historical interpretation of the Gospel arose. They were not added to guard against the heresies of the day, but to defend an innovation. The Apostles' Creed alleges seven historical facts about Jesus Christ the son of the Creator, his birth, crucifixion, burial, resurrection, ascension, session at the right of God, and second coming, and all the seven emphasize the reality of the life of Jesus as against the older view. The truth is that the Old Roman Symbol and the Apostles' Creed are evidence that the church had lost the faculty of spiritual vision and had become a prey to the besetting sin of all ecclesiasticisms—the worship of the letter.

What confirms us in all this is the fact that when we go back to the first century we find that there were other communities or churches besides those which were organized around the tradition of a historical Jesus. The Epistles of Paul are evidences of this fact. It is impossible to believe that the churches or communities to whom Paul preached his view of a spiritual Christ or Messiah revealed to him by his own ecstatic experiences or visions were derived from the church of Jerusalem of which Peter and James and John were the founders and which was organized around the story of a historic Jesus. Paul was at variance with Peter and James and John whom he called "pillar apostles" not in a very complimentary way. In the letters of Paul we are introduced to communities or churches entirely different from those which took the synoptic Gospels as their inspiration and guides.

Paul does not follow the synoptic tradition at all; he follows a Christ of his own and speaks of his own gospel. To Paul the views of the "pillar apostles" seemed decidedly materialistic. It is indeed difficult to believe that there was any such record of the life and teaching of Jesus in existence as the synoptic Gospels contain in the possession of the church at Jerusalem; for with an authority such a record would imply, how could Paul have had any chance of successfully withstanding the "pillar apostles," or of persuading the communities or churches formed by them to leave them and follow him? The immense probability is that both "Jesus"

and "Christ" were divine names before the Christian era, that both were equally unhistorical, and that they were brought together as denoting a single being by the movement that afterwards became historical Christianity. Whether this be so or not, Paul's Epistles bear witness to the existence of churches or communities which had been long in existence when Paul visited them. Paul's words and phrases are the same as those in use in these communities or churches; they knew what he was speaking about, so that he did not need to define his terms. Paul had no affinity with churches based upon the tradition of a historical Jesus such as we have in the synoptic Gospels; but he has a very close affinity with those other churches or communities whose members believed in a mystic Christ and whose technical terms were all borrowed from Gnosticism which recent research has proved to be pre-Christian.

Paul was not converted to belief in a historical Jesus. He was changed from being an official persecutor of the Messianic sects to a preacher of a mystic Christ or spiritual Messiah, the conception of which, he declares, he did not derive from man; that is to say, the Christ he preached was born of his own immediate experience and revelation. He got a chance of a hearing for his spiritual gospel because it was on a level with the belief in Jesus. If one had been historical and the other not, he would have been as one beating the air. What emerges clear as daylight is that the churches or communities he founded, as well as those he found already established when going on his missionary journeys, were not communities which believed in a historical Jesus; they were of a mystical nature resembling the Therapeutae of whom Philo tells us in his "On the Contemplative Life"—people devoted to the cultivation of the life of contemplation and of union with God. It is not an unlikely supposition that it was with some one of those communities that Paul spent his three years after his conversion, and that it was the light and inspiration he received from that source which emboldened him to be the apostle he afterwards became. It is here doubtless that we are to find the oldest form of the Christian faith. What we have in the synoptic Gospels is a teaching decidedly lower in spiritual insight and tone than that current in the mystical sects to which Paul ministered. They believed in a Saviour who was a heavenly being; belief in the Logos was a fundamental part of their creed; and if there was a historical Jesus at all, the great probability is that he was a member of one of the mystic sects of which the age was full. It is extremely unlikely

that the historical Jesus shared the ignorant views of the people as the synoptic Gospels represent him as doing. The probability is that in these representations of the synoptic Gospels we have just that kind of misunderstanding which always takes place when a mystical teacher attempts to communicate truth to people on a lower level of life and experience. The supreme misunderstanding was the identification of the historical Jesus with the mystical Christ, the Logos. The real Saviour of men, as the real Jesus would doubtless have been the first to declare, is not a historical person, but a divine being who dwells in the soul as Paul teaches. This belief in a mystic Christ, in a heavenly being, in a divine Logos, long antedated the beginning of our era, both in Jewry and among the Greeks.

Now just as those mystic sects of the first century and before, represented by Paul, embodied a more spiritual conception of religion and of life than that embodied in the synoptic Gospels, so the communities or churches of the second century represented a phase of Christianity that was different from, and opposed to, that taught by the growing and triumphant Catholic church. And it is to be noticed further that this form of Christianity was the original one and that the Christianity of the Catholic church was a development of that. This original form of Christianity is known as Gnosticism. All the apocryphal Gospels and Acts are saturated with Gnosticism. Here is where recent investigations into the genesis and development of Gnosticism help us greatly to discover the first form of the Christian faith. Instead of Gnosticism being an outgrowth of Christianity as has long been supposed—a heresy which was persecuted and finally expelled out of existence—the various forms of Gnosticism, Jewish and Christian, of the early centuries, were only particular cases within a movement that included much more. The apocryphal Gospels and Acts tell us what Gnosticism was much better than the school dogmas of Basilides and Valentinus which we know only through the reports of their ecclesiastical enemies, because they formed the main means of Gnostic public propaganda. There was a very wide circulation of such Gospels and Acts in the second century. They are deeply spiritual in their meaning though the outward form was often fantastic and grotesque enough. But we must remember that it is only to the modern mind that they seem fantastic and grotesque. They were not so to the men of the second century; to every shade of mind of that age they were equally and entirely credible. And it was not the mythical and legendary element that offended the orthodox party of the day; it was the inner

spiritual teaching, and that they assailed with misrepresentation, and tried to overwhelm with ridicule. It is just to say that we of this age would be repelled by the marvelous nature of the stories they relate. The apocryphal Gospels and Acts which embody the inner spiritual teachings of those Gnostics read to us like wild romances, but to them they symbolized actual occurrences of the inner life, facts of direct spiritual consciousness. The teaching is for those who knew the nature of the inner life by direct experience; for all others they were foolishness. We have the principle stated by Paul in his letter to the Corinthians: "The natural man perceiveth not the things of the spirit of God, for they are foolishness unto him; neither can he know them, for they are spiritually discerned."

It has always been a mystery how such wild imaginings and learned subtleties as the doctrines of the Gnostics seem to be (as represented by the orthodox Church Fathers) could make any deep impression on the minds of men of that age or indeed of any age. The mystery is explained when we turn our attention to the popular literature of the movement as embodied in the apocryphal Gospels and Acts of the second century; and when, especially, we are able to look below the surface and discern the inner spiritual meaning of the narratives. They were so popular in the second century that they could not be disposed of by ridicule simply, and the orthodox Church Fathers had to have recourse to other means to meet them. It is because of this fact that we have these apocryphal Gospels and Acts at all. The orthodox Church Fathers boldly adopted the most popular narratives from the heretical books, and after carefully eliminating what they deemed the "poison of false doctrine" replaced them in this purified form in the hands of the people. Fortunately for us this purification has not been complete, and some of the "poison" has been preserved. Many things of great beauty are found in these Gospels and Acts amid much that seems fantastic and grotesque. But, as I have said, they are so because we do not possess the key that will open up the meaning. This key is found in the man-mystery, the man-myth, or man-doctrine, which was central in all the mystery institutions of antiquity.

Briefly put, it is the story of the descent of man from his heavenly home and his return to that state of glory after having mastered the powers of the world. There is nothing so ancient as this doctrine; it is lost in the mists of antiquity, and in the centuries immediately preceding the beginning of our era it was a well-devel-

oped doctrine in the whole Graeco-Roman world. It was the jealously guarded secret of every mystery institution of antiquity. The whole ancient world was honeycombed with these mystery institutions. They were practically universal, being found in Chaldea, Phenicia, Palestine, Egypt, Phrygia and Greece; and in every one of them the central doctrine was this myth or mystery of man. In Plato, whose writings were the Bible of the Greeks, we have allegory upon allegory describing the soul of man in his heavenly home. The state of man in this world these Gnostics called a state of death. We have a hint of this in Paul's letter to the Colossians where he describes man as "dead" and his "life as hid with Christ in God" (Col. iii. 3). He means that the true life of man is buried in matter and awaits resurrection, which does not mean resuscitation of a dead body, but the awakening of the spirit of man into consciousness of its divine life. In the Epistle to the Ephesians the apostle quotes a part of a Gnostic hymn: "Awake thou that sleepest, and arise from the dead, and the Christ shall shine upon thee," and this does not mean a call upon dead bodies to come out of graves, which would be absurd, but a call upon the spirit to awaken out of its state of unconsciousness and realize its true life. With the Gnostics of the apocryphal Gospels and Acts the death of Jesus was the symbol of a profound experience which the individual spirit must pass through on its upward journey as a condition of its further advancement. The resurrection of Jesus they similarly looked upon as a symbol of the new birth of illumination of the spirit, its coming to life from its previous death state. The instruction given in the apocryphal Gospels and Acts, as well as in many Gnostic treatises, such as *Pistis Sophia,* is represented as having been given by Jesus to his disciples after his resurrection, which means that the truth taught is what the soul sees in its state of illuminated consciousness. The germ of the Christ life, the spark of divinity which the poet Browning says "disturbs our clod," the image of God all men bear, the light which every man brings with him into the world, must descend into matter; the "dead" with the Gnostic writers and with Paul are those in whom the consciousness of the divine has not been awakened. Resurrection is the awakening of this germ to life. This is the real resurrection of which the historicized rising from the dead of the body of Jesus the canonical Gospels speak of is a symbol. The story of the descent of the soul into matter, its gradual conquest of matter, its awakening to its true life, and its return to its former state having mastered

the powers of the world is the myth of man found in all the ancient mysteries. It is the same story which the New Testament tells in the form of a symbolic life.

Now from a human point of view it was necessary that such a form of Christianity should not become the Christianity of the church. For very soon came the fall of the Western Empire and the inrush of the barbarians from the north. Very soon a wild sea of savage tribes surged and heaved where once the cultured fields of the Old World had been. It was impossible that the strong virile minds of Goth, Hun, and Vandal could comprehend the religion that satisfied these philosophers of the East. A cruder faith was needed and a cruder faith became the faith of the Catholic church. The purer faith became a heresy and was bitterly opposed by the dominant church. As the Catholic church grew in power it grew too in priestly claim and in arrogance. Even as early as the latter part of the second century it had become a visible hierarchy. We find Irenaeus uttering the famous dictum that where the church is —and even as early as his day the church was a visible organization with its clergy and sacraments—there is the spirit of God, and where the spirit of God is there is the church.

A proud, arrogant, ambitious church in the course of its history has been guilty of many crimes, but perhaps the blackest record its history can show is its persecution of these Gnostics. These people had a long ancestry. "The method of history," says Prof. G. P. Fisher of Yale, "is never magical. In proportion to the magnitude of the event are the length of time and the variety of agencies which are employed in producing it." Professor Fisher applies this remark to the Reformation of the sixteenth century, showing that "never was a historical criticism more elaborately prepared for, and this through a train of causes which reach back into the remote past." But the words apply specially to the advent of Christianity; for as a matter of fact Christianity was in the womb of the pagan world for centuries. As Gnosticism was the child of paganism, so Christianity was the child of Gnosticism. The words of St. Augustine are strictly true. "The very thing which now is called the Christian religion existed among the ancients, nor was it absent in the beginning of the human race before Christ came in the flesh, since when the true religion which already existed began to be called Christian." In the nicknames which the heresy hunters of the time hurled at the Gnostics we have a clue to the question whence they derived their teaching. The orthodox Church Fathers

were neither sparing nor nice in the names they applied to the Gnostic heretics—devils, snakes, hounds, wolves, vipers, and first-born of Satan. These names of course do not give us much light except upon those who used them; but when less thoroughly aroused with theological passion, and consequently in less bitter mood, they said that the Gnostics derived their teaching from Pythagoras and Plato and Heraclitus and Cleanthes, and from the mystery institutions of Greece, Egypt and the East generally. This was the truth, but instead of being a reproach it was their glory. This meant that the teaching was the best in the religious teaching of the ancient world. Instead of coming into a world of universal darkness with its one divine light of truth, Christianity came from the same source as Gnosticism. In the Epistles of Paul we have echoes of what was taught in Egypt and Greece two or three hundred years before. There is nothing of which we are so sure as the existence of a well-developed and well-defined doctrine in the Hellenistic world of the first centuries before Christ, of the descent of man from the heavenly or archetypal man, and of his return to pristine glory with the experience he has gathered from his contact with, and conquest of, the world of matter and form. This Paul calls the "mystery" of Christ, the mystery hid from ages and generations, but now made manifest.

The story of a Christ who was the Saviour of the world, the divine man who was the representative of a great spiritual process, the mediator between God and men, the ideal man who was overcome in his struggles for human salvation but conquered in being overcome, is the story which the world has repeated to itself over and over again. It is not original with the New Testament, every feature of it was familiar to those who were initiated into the mysteries. This should be enough to show us that we are not in the presence of literal fact. There is no doubt that the crucifixion as Paul conceived it had cosmic significance—it is not merely the death of a martyr. The center and soul of the gnosis of the ancient world was the Cross. The technical phrase for it among the Gnostics is one used by Paul, the "cross the power of God." Wherever the gnosis had established itself the kernel was the cross. It is obvious that in these places it could not mean the death of Jesus for that was a local happening. It meant the great world-passion, the sacrifice of God in the creation, Deity laying down his life in the universe of matter and form. And to Paul the cross was the symbol of this heart-moving conception.

The interpretation of Paul's determination not to know anything among men save Jesus Christ and him crucified that makes him mean to refer to a series of historic facts eviscerates it of all real content. In the creation was the Calvary of Deity. The cross is thus the background plan of the universe. To know the cross from this higher standpoint is to know all there is to know; there is nothing beyond this. The cross was the symbol of a profound mystery which opened up the heart of Deity himself to the gaze of the world. The divine sufferer was God himself, who in creating the universe sacrificed himself for it. The cross, therefore, represents the greatest of all sacrifices, not something that happened once and once for all, but something that is eternal and timeless—the sacrifice of God in and for his own creation that could not be unless he poured his own life into it and restricted himself within the forms of matter. "Confessedly great is the mystery of godliness." Unthinkable in its magnitude is this sacrifice, for it means nothing less than the identification of the infinite with the finite in its lowest forms. Here is the profoundest mystery open to human contemplation to speak of which is possible only in forms of symbol and parable. The literal truth is too vast, too mysterious, too sublime, to be made known to human comprehension. It is the mystery before which angels, we are told, veil their faces; and to gain a single glimpse of it one may well surrender all other knowledge and determine, as Paul did, to know nothing else. Here is the oldest form of the Christian faith. The story of Jesus is the parable of this infinitely larger truth. It is the symbol of the Lamb slain from the foundation of the world, that is, prior to human history, the emblem of divine body and blood voluntarily sacrificed in outward physical nature and entombed in the lower consciousness of man. It was the claim of the second century Gnostics that Christianity was none other than the consummation of the inner doctrine of the mystery institutions of all the nations; and it is this interpretation of the Gospel story which is set forth in the apocryphal Gospels and Acts. The end of them all was the revelation of the mystery of man which is none other than the mystery of Christ.

DUNDEE, SCOTLAND. K. C. ANDERSON.

HAMILTON'S HODOGRAPH.

In Mach's *Mechanics* the space devoted to the hodograph of Sir William Rowan Hamilton is barely a page, half of which is

taken up by the diagram; the diagram, too, is not drawn strictly according to the construction of Hamilton, but according to the usual manner of present-day text-books. This seems to me to be something of an injustice to an exceedingly brilliant piece of work; for from it and Hamilton's theorem of the isochronism of the circular hodograph can be deduced the more important properties of motion in orbits described to a center of force according to the Newtonian law, Lambert's theorem for the area of an elliptic sector in terms of the bounding *radii vectores* and the chord of the sector, and Euler's corresponding expression for the area of a parabolic sector. The proofs are exceedingly simple and demand very little knowledge of the properties of conics, and no calculus or coordinate geometry at all. For other interesting and elegant applications of the hodograph, reference should be made to Tait's *Quaternions* and papers in the *Proc. R. S. E.*, and to Tait and Steele's *Dynamics of a Particle*. But the matter which follows should, I think, be sufficient to corroborate the opinion expressed above.

I. The first use of the curve is ascribed to Bradley,[1] and it is probably his definition that is generally given in elementary text-books, where its only use is to obtain the acceleration towards the center of a particle moving in a circle with uniform speed.

DEF. 1. If a point be in motion with any velocity in any orbit, and if at any instant a line be drawn from a fixed point representing on some chosen scale the velocity of the point at the instant in magnitude and direction, the locus of its end is the "hodograph" of the motion.

Not even in the particular case of motion in a circle does this, the usual definition, so readily demonstrate the connection between the hodograph and the orbit as the definition originally given by Hamilton in a paper communicated to the Royal Irish Academy in 1846,[2] which is as follows:

DEF. 2. "If, in an orbit *round a center of force*, there be taken on the perpendicular from the center on the tangent at each point a length equal to the velocity at that point of the orbit, the extremities of these lengths will trace out the hodograph."

Note. The use of the word "equal" does not introduce any difficulty, unless we attempt to verify the formulas obtained by reference to the theory of dimensional units. It is to be noted that

[1] Sir Robert Ball, Article on "Gravitation," *Encyc. Brit.*
[2] *Proc. Roy. Ir. Acad.*, 1847.

the definition of Hamilton is more specialized than the other, referring only to central orbits.

II. Newton showed that for any central force the areas swept out by the radius vector are proportional to the times.[3] The following is a simple proof of this theorem.

If O is the center of force, and P, Q, R points on the orbit separated by *very small unit intervals* of time, and MQN, PM, RN are drawn perpendicular to OQ, MQ, QN respectively; then MQ, QN measure the velocities transverse to OQ, before and after the position Q.

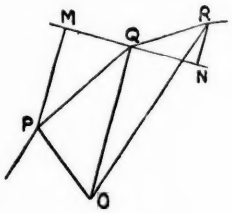

Fig. 1.

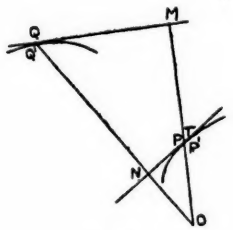

Fig. 2.

Since the force is central, $MQ = QN$,

$$\therefore \triangle OPQ = \triangle OQR;$$

and hence the proposition follows immediately.

It follows from Newton's theorem that, if the rate at which the area is swept out is denoted by $\frac{1}{2}h$, and the perpendicular from O on the tangent at P by p, then $pv = h$.

Hence, in Fig. 2, $ON \cdot OQ = h$, and Q is the image of N in a circle whose radius is $\sqrt{h}$; also, if P', Q' are near points to P, Q, and the tangents at P, P' meet in T, then Q, Q' are the poles of two lines passing through T, and therefore QQ' is the polar of T. That is, the tangent at Q is the polar of P.

The hodograph is therefore the polar reciprocal of the orbit.

It is also evident that $OM = h/OP$; or, in other words, the product of the length of the perpendicular from the center on the tangent to the hodograph and the length of the corresponding radius vector of the orbit is constant $(= h)$.

Again, since the tangent at Q is the direction of the velocity of Q in the hodograph, this velocity is perpendicular to the radius

[3] Newton, *Principia*, Book I, Prop. 1.

vector. The hodograph given by the usual text-book definition is therefore identical with that given by Hamilton's definition, when turned through a right angle.

Further, since OQ, OQ′ represent the velocities at P, P′ turned through a right angle, it follows that QQ′ represents the change of velocity between P and P′ when turned through a right angle. Hence the radial acceleration of P is measured by the velocity of Q in the hodograph.

IV. The angle between the normals at QQ′ is equal to the angle between the radii OP, OP′.

Hence in Fig. 3 we have

$$QQ'/\rho = PN/r = 2\triangle OPP'/r^2;$$

and if the acceleration in the orbit is denoted by f, then

$$f/\rho = h/r^2.$$

This formula for the radius of curvature of the hodograph solves immediately the case

$$f \propto 1/r^2.$$

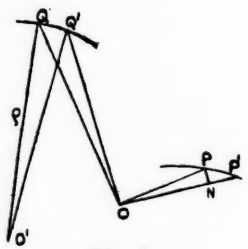

Fig. 3.

For, if $f \propto 1/r^2$, ρ is constant, and the hodograph is therefore a circle, and its polar reciprocal is a conic section with the pole as a focus; conversely, if the orbit is conic, with a center of force at a focus, the polar reciprocal is a circle, ρ is constant, and the law of force is $f \propto 1/r^2$.

The feet of the perpendiculars from the focus to the tangents to the orbit lie on the auxiliary circle (the tangent at the vertex in the case of the parabola). Hence the hodograph is an inverse of the auxiliary circle or the tangent at the vertex in the case of the parabola. It follows that the focus is within, on, or without the hodograph, according as the orbit is an ellipse, parabola, or hyperbola.

On account of the fact that the hodograph of an orbit described under the Newtonian law, $f \propto 1/r^2$, is a circle, Hamilton designated the law as the "Law of the Circular Hodograph."

V. If the law is $f = \mu/r^2$, the diameter of the hodograph is easily seen to be $2\mu/h$. Also, the diameter through O of the hodograph corresponds to the diameter aa' of the orbit and is similarly divided at O.

In Fig. 4, let A, B, A′, B′ correspond with the tangents at a, b, a', b', and therefore OA, OB, OA′, OB′ represent the velocities at the points a, b, a', b'.

Now, $aa' = Oa + Oa' = h(1/OA + 1/OA') = h.AA'/OA.OA'$.

If, then $aa' = 2a$, we have $a = h\rho/OB^2$; i. e., $OB^2 = h\rho/a = \mu/a$.

Therefore, the velocity at the end of the minor axis of the eclipse

$$= \sqrt{\mu/a}.$$

The velocity at any point of the orbit is now at once obtainable.

CASE I. When O is within the hodograph, as in Fig. 5.

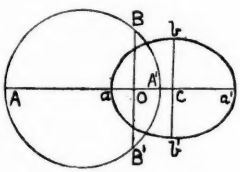

Fig. 4.

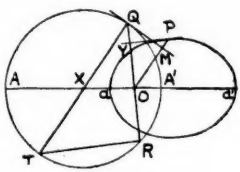

Fig. 5.

If QO and QX meet the hodograph a second time in R and T, the triangles OQM, QTR are similar;

$$QT : OQ = QR : OM,$$

$$\therefore 2\rho.OM = OQ.QR = OQ^2 + QO.OR = OQ^2 + AO.OA';$$

$$\therefore OQ^2 = 2\rho h/r - \rho h/a \qquad \text{(from § IV)}$$

$$\text{i. e., } V^2 = \mu(2/r - 1/a).$$

CASE II. When O is on the hodograph, $V^2 = 2\mu/r$.

Case III. When O is without, $V^2 = \mu(2/r + 1/a)$.

VI. A most remarkable theorem connected with the hodograph was communicated to the Royal Irish Academy in March, 1847, without demonstration,[4] by Hamilton; he however furnished a proof by quaternions in 1853.[5] The following simple geometrical proof arises from hints given by Hamilton's proof.[6]

THEOREM. "If two circular hodographs, which have a common chord passing or tending through a common center of force, be both cut at right angles by a third circle, the times of hodographically describing the intercepted arcs will be equal."

[4] Cayley's report to the Brit. Assoc., 1862.

[5] Hamilton, *Lectures on Quaternions*, 1853.

[6] Professor Blyth, Article on "Hodograph" in *Encyc. Brit.*

If ρ is the radius of the hodographic circle in Fig. 6, and the law of force is $f=\mu/r^2$, then, by § IV, we have

$$\rho h = fr^2 = \mu \text{ (a constant)};$$

also, by § III, we have $r.\mathrm{OM}=h$, and therefore

$$t = \mathrm{PQ}/f = \mu.\mathrm{PQ}/\rho^2.\mathrm{OM}^2.$$

Now the angles PXQ, RVR′ are equal and small,
$\therefore \mathrm{PQ} = \rho.\sin \mathrm{RVR}' = \rho.\mathrm{RR}'.\sin \mathrm{VRR}'/\mathrm{VR}' = \rho.\mathrm{RR}'.\mathrm{OM}/\mathrm{OR}.\mathrm{PR}$
ultimately; and, from the similar triangles, XPL, ROM,

$$\rho.\mathrm{OM} = \mathrm{OR}.\mathrm{PL};$$

$$t = \mu.\mathrm{RR}'/\mathrm{OR}^2.\mathrm{PL}.\mathrm{PR} = \lambda/\mathrm{PL}.$$

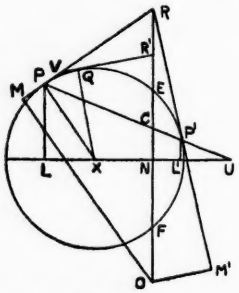

Fig. 6.

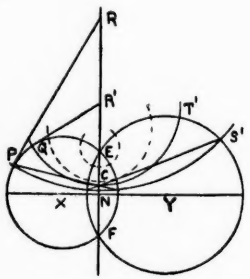

Fig. 7.

Similarly, for the description of the arc P′Q′, we have $t=\lambda/\mathrm{P}'\mathrm{Q}'$,

$$t+t' = \lambda(1/\mathrm{PL}+1/\mathrm{P}'\mathrm{L}') = 2\lambda/\mathrm{CN}.$$

But any two circles, intersecting in EF, are cut orthogonally by a system of circles having their centers on EF produced in either direction. If we start with the orthogonal PSP′S′ in Fig. 7, C is the radical center of the three circles whose centers are R, X, Y; and hence, since the times of describing arc PQ+arc P′Q′ and arc ST+arc S′T′ are each equal to $2\lambda/\mathrm{CN}$, these times are equal. Therefore, by "filling up" the arcs PP′, SS′ with a system of orthogonals, it follows that the times of description of the whole arcs PP′ and SS′ are equal.

VII. Let us consider the whole system of coaxal circles passing through E and F, and their corresponding orbits.

If the diameters of the hodographs passing through O are drawn, the products of the segments into which they are divided

at O are respectively equal to μ/a_1, μ/a_2, μ/a_3,...., where a_1, a_2, a_3, ...are the semi-major axes of the orbits, when O does not coincide with either E or F. Hence, since O is on the radical axis of the system, it follows that the major axes are all equal.

If O coincides with E or F, and $4a$ is the latus-rectum of one of the parabolic orbits, $2\rho . a = h$, or $a = h^2/2\mu$.

Again, any one of the hodographs can be completely traced out by a set of orthogonal circles whose centers traverse the whole of the radical axis except the part EF intercepted within the hodographs. Therefore, by Hamilton's theorem, the periodic times are all equal, for each separate position of the point O.

Further, if the orbits are supposed to have their major axes all in one straight line, so that O is the middle point of EF, one of the orbits is the circle of radius a, which also passes through the ends of the minor axes of all the orbits. The radius of its hodograph is equal to the velocity at the end of a minor axis of the general orbit of the system, and is equal to $\sqrt{\mu/a}$. Also, the velocity in this hodograph is equal to the acceleration in the circular orbit, i. e., is equal to μ/a^2. Therefore, we have

$$\text{Periodic Time} = T = 2\pi\sqrt{\mu/a} \div \mu/a^2 = 2\pi\sqrt{a^3/\mu},$$

for the whole system of orbits.

VIII. Hamilton pointed out that his theorem of isochronism was the same as Lambert's theorem. This is not generally taken to mean the detailed elliptic quadrature theorem as given in Williamson's *Calculus* and elsewhere, but the central force theorem thus enunciated:

In elliptic orbits of equal major axes, described in the same periodic time, under the action of a central force to a focus, the times of describing any sectors are the same, if the chords of the sectors and also the sum of the bounding radii vectores are the same.

A geometrical proof of the theorem in this form was given by H. R. Droop,[7] who pointed out that Hamilton had not, to his knowledge, previously published any demonstration of it; a short analytical proof of Hamilton's theorem is to be found in Tait and Steele's *Dynamics of a Particle*, with the remark, "It will readily be seen that this is in substance the same as Lambert's theorem." On the other hand, Cayley gave an analytical proof of Hamilton's theorem by assuming that of Lambert.[8]

[7] *Quart. Journ. Math.*, Vol. I, pp. 374-378.
[8] *Phil. Mag.*, 1857, pp. 427-430.

The following simple demonstration is practically the same as that of Droop, with the slight improvement that the connection, with lines in the hodograph, of the sum of the *radii vectores* and of the chord of the sector is plainly shown.

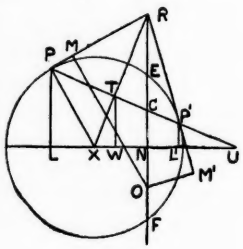

Fig. 8.

In Fig. 8, since the triangles XPL, ROM are similar, as are also XP'L', ROM',

$\therefore r+r' = h/\mathrm{OM} + h/\mathrm{OM}'$,

$= \rho h(1/\mathrm{PL} + 1/\mathrm{P'L'})/\mathrm{OR}$,

$= 2\mu/\mathrm{OR}.\mathrm{CN}$;

hence $r+r'$ is constant.

Again, if δ is the chord, p the perpendicular on it from the focus, and θ the angle between the radii in the orbit which is equal to the angle at the center of the hodograph, then, since R is the pole of the chord, $p.\mathrm{OR} = h$;

$\therefore \delta = rr' \sin\theta/p = h\mathrm{OR} \sin\theta/\mathrm{OM}.\mathrm{OM}' = \mu\rho \sin\theta/\mathrm{OR}.\mathrm{PL}.\mathrm{P'L'}$.

But $\mathrm{PL}.\mathrm{P'L'} = \mathrm{TW}.\mathrm{CN} = \mathrm{CN}.\mathrm{NR}.\mathrm{XT}/\mathrm{XR}$,

and $\rho \sin\theta = 2\mathrm{XT}.\mathrm{PR}/\mathrm{XR} = 2\mathrm{PR}.\mathrm{XT}/\mathrm{XR}$;

$\therefore \delta = 2\mu.\mathrm{PR}/\mathrm{CR}.\mathrm{CN}.\mathrm{NR} = $ a constant.

Note. $r+r' : \delta = \mathrm{NR} : \mathrm{PR}$.

IX. Although no loss of generality is introduced in the last part of § VII by the assumption that the axes of the orbits all lie in one straight line, there will be a loss of generality if we attempt to work out the time of description for a sector on the same assumption; for the inclusion of the circle whose center is O determines the value of $r+r'$ to be equal to $2a$. In this case the chord must be either parallel to the major axis of any orbit or a diameter of it; and the latter is excluded from consideration by Hamilton's theorem, since it is impossible for an orthogonal circle to cut a hodographic circle in points on opposite sides of the diameter perpendicular to the radical axis. When, however, O is not taken to be the middle point of EF, all positions of the *radii vectores* are included; and one of these positions is that in which the chord is parallel to the minor axis.

From Fig. 9, it is evident that area of $p\mathrm{C}q\mathrm{S} = \mathrm{CS}.p\mathrm{N}$, and area of $p\mathrm{C}q\mathrm{A} = a^2 \sin^{-1} y/b$;

$$\therefore \text{ area of PSQA} = ab\,(\sin^{-1} y/b - ey/b).$$

Similarly, in the case of the parabola, it is seen from Fig. 10 that, since area of segment PAQ = ⅔ area of $\propto$ PAQ,

$$\therefore \text{ area of sector PSQA} = y(x+3a)/3.$$

X. From the results obtained in § IX, we can show in a very simple manner that not only does Hamilton's theorem contain that of Lambert in its detailed form, but also the corresponding theorem of Euler for a parabolic sector.

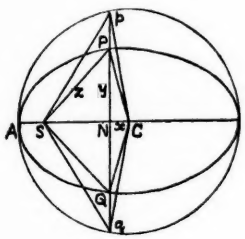

Fig. 9.

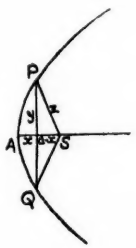

Fig. 10.

LAMBERT'S THEOREM:

Assume $\phi - \phi' = 2\sin^{-1} y/b$ and $\sin\phi = \sin\phi' = 2ey/b$;

then, since $\sin\frac{1}{2}(\phi-\phi') = y/b$, $\therefore \cos\frac{1}{2}(\phi-\phi') = x/a$;

also since $2\sin\frac{1}{2}(\phi-\phi')\cos\frac{1}{2}(\phi+\phi') = 2ey/b$,

$$\therefore \cos\tfrac{1}{2}(\phi+\phi') = e, \quad \therefore \sin\tfrac{1}{2}(\phi+\phi') = b/a;$$

$$\cos\phi' + \cos\phi = 2\cos\tfrac{1}{2}(\phi+\phi')\cos\tfrac{1}{2}(\phi-\phi') = 2ex/a,$$

$$\cos\phi' - \cos\phi = 2\sin\tfrac{1}{2}(\phi+\phi')\sin\tfrac{1}{2}(\phi-\phi') = 2y/a.$$

Now, referring to Fig. 9, if SP is equal to z, $a - ex = z$, and therefore $ex/a = 1 - z/a$; hence

$$\cos\phi = 1 - (z+y)/a, \text{ and } \cos\phi' = 1 - (z-y)/a;$$

and these expressions only contain z, y, a, which are equivalent to $\frac{1}{2}(r+r')$, $\frac{1}{2}\delta$, a; and these are invariable. The time of description being also invariable as well as the period for the whole orbit, we can say that in general,

Area of elliptic sector $= \frac{1}{2}ab[\phi - \phi' - (\sin\phi - \sin\phi')]$, where

$4\sin^2\frac{1}{2}\phi=(r+r'+\delta)/a$ and $4\sin^2\frac{1}{2}\phi=(r+r'-\delta)/a$.

EULER'S THEOREM:

By VII, $h^2=2a\mu$; and the area described $=ht=\sqrt{2a\mu t}$; hence

$$t=y(x+3a)/3\sqrt{2a}\mu=2\sqrt{x}(x+3a)/3\sqrt{2\mu};$$

and, assuming $\lambda^2=z+y$, and $\mu^2=z-y$, we have

$$2(a+x)=2z=\lambda^2+\mu^2$$

$$2(a-x)=2\sqrt{(z^2-y^2)}=2\lambda\mu.$$

Therefore, $2\sqrt{x}=\lambda-\mu$, $x+3a=\lambda^2+\lambda\mu+\mu^2$

and $t=(\lambda^3-\mu^3)/3\sqrt{2\mu}$.

Now, t, z, or $\frac{1}{2}(r+r')$, y or $\frac{1}{2}\delta$, and therefore λ and μ, are all invariable; hence, in general, since area equals $\sqrt{2a\mu t}$, we have

$$\text{Area of parabolic sector}=\sqrt{a}(\lambda^3-\mu^3)/3,$$

where $2\lambda^2=r+r'+\delta$, and $2\mu^2=r+r'-\delta$.

Note. The sign of μ changes when PQ cuts AS produced.

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MONISM AND THE ANTINOMIES.

In the following sketch it is the purpose of the author to offer a few brief suggestions with a view to clearing up some of the long-mooted problems of philosophy and theology, particularly in regard to the so-called philosophical antinomies. Of these Kant has enumerated four, viz., (1) whether the universe is temporal and finite, or eternal and infinite; (2) whether composite substances are capable of resolution into irresolvable simple parts, or whether "naught that is simple exists"; (3) whether causality is the sole cause of phenomena or whether free will must be considered along with it; and (4) whether the world (i. e., the sum-totality of existence) possesses or does not possess some form of necessary existence. The contention of Kant is that these paradoxes do not exist, save in our own thought, and that it is due to the limitation of our mental powers that we seem to see such antinomies. With regard to the most important of these, namely, whether causality is the sole cause of existence, he has demonstrated absolutely, and with

consummate skill, the perfect compatibility of inexorable necessity with free will; and while it would appear that he has not demonstrated conclusively the existence of free will in itself, yet he gives the latter a presumptive status so strong that we are practically obliged to admit that it is part and parcel of the scheme of things and must therefore be considered at length in all serious philosophical inquiry.

To these metaphysical antinomies of Kant there must be added the theological antinomies of Mansel, which we cannot better set forth than by quoting the following from Mansel, as given in Spencer's *First Principles* (p. 35):

"How....can Infinite Power be able to do all things, and yet Infinite Goodness be unable to do evil? How can Infinite Justice exact the utmost penalty for every sin, and yet Infinite Mercy pardon the sinner? How can Infinite Freedom be at liberty to do or to forbear? How is the existence of Evil compatible with that of an infinitely perfect Being; for if he wills it, he is not infinitely good; and if he wills it not, his will is thwarted, and his sphere of action limited?"

All of these, it will be seen at a glance, have to do directly with the antinomy of free will. Besides, their pragmatic interest is of highest importance; and, being apparently in direct paradox to the doctrine of monism, which maintains the essential oneness and unity of everything, they must receive more detailed consideration here.

The first two antinomies of Kant, relating to the infinitude of the universe and to its complexity, have little direct bearing on monism, as far as our purposes are concerned, since it is necessary that all parts of an entirety, whether finite or infinite, or whether capable of resolution into simple parts, or incapable of such analysis, must bear certain unalterable relations to one another, since that is what the concept of entirety implies.

His third antinomy offers greater difficulty. If causality were the sole cause of everything, it would be necessary that monism should obtain. On the other hand, if causality is not the sole cause, but free will must be considered along with it, then even if free will can only modify things spontaneously along the lines or within the bounds which necessity prescribes, a higher law must be shown to exist, or at least its possibility must be shown to be reasonable, as an intellectual conception, before we can include it (i. e., free will) alongside of causality in a complete body of monistic doctrine.

Here we will add, as giving an important side-light on this

question, that due regard must be paid to the general consensus of human kind, to the effect that there is an essential moral element in a certain class of actions (lying within the scope of ethics)—a conception which thousands of years of philosophical thought and dispute have failed to shake.

The notion of a spontaneous cause, infinite in its operation and in its scope of activity, is not difficult to form; and the presumptive evidence which militates so strongly in favor of the doctrine of free will in human agents, must carry with it, as a necessary corollary, the doctrine of a freedom, not finite as with man, but absolutely infinite.

That such an infinite freedom must be rational, goes without saying. For an infinite being, omnipotent and omniscient, to err, even in a moral sense, were an utter absurdity. We should, therefore, presumptively ascribe to this infinite freedom a higher law of freedom, transcending all concept of natural law, just as the concept of infinite freedom, on the other hand, transcends all our ideas of the limited freedom possessed by man. In addition, we must not think of natural law as a limitation upon the infinite, but as part and parcel—an integral portion so to speak— of its higher law of liberty. And this higher liberty of the infinite is to be considered, theologically, as the goal which the regenerated human soul ever strives to attain through an infinite series of spiritual progressions.

The difficulty in regard to the inabilty of the absolute, infinitely good, to do evil, must be regarded in the light of an inscrutable problem. Of course, it is only moral evil of which we here speak. To infer that since God created man, and man sins, God must be the author of evil, were well-nigh blasphemous. Man attains the privilege of grace only by the exercise of his divine prerogative of freedom, and whenever he abuses it *he* becomes the author of evil.

The reconciliation of infinite justice with infinite mercy should be an easy matter. It is not reasonable to suppose that the justified sinner escapes the natural penalty of his sins, but rather that he escapes the added penalty that would be inflicted for his continuance in the way of evil. Justification when used in a theological sense means exactly what that word naturally implies,—that the saved sinner is justified, because in the agony of his contrition and in the diminished possibilities of future happiness which he has already incurred, he has, at the moment of conversion, paid the full

penalty for his sins. The exercise of the divine prerogative of mercy is thoroughly at one with the higher law of absolute freedom.

"How can Infinite Freedom be at liberty to do or to forbear?" Because it is *Infinite Freedom.*

And as to the existence of moral evil being a limitation of the freedom of the absolute, as some one has well said, "a self-imposed limitation is no limitation at all," and we might add that God, in sharing his prerogative of freedom with man, diminishes his own limitation not one whit, because what is infinite in the first instance is incapable of diminution.

The conclusion to be arrived at from a more detailed consideration of this theme is as follows: That the universe of material things and forces is a totality, but not the totality of all existence; that the freedom of man is consistent through and through with the universality of natural law; that the existence of an absolute being is in no way incompatible with the existence of other entities, of a finite character; and lastly, that the sum total of material things and forces, of man and his actions, of mind and its thoughts, of freedom and its prerogatives and privileges, each and every one of them, together with the absolute, are woven together, as it were, in conformity with a higher law transcending all natural law,—all in a comprehensive, necessary, consistent and perfect monistic scheme of existence.

CARL V. POLAND.

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KANT'S ANTINOMIES AND THEIR SOLUTION.

Kant's antinomies have proved a puzzle to thinkers. The great author of the *Critique of Pure Reason* believes that there are some statements concerning which the affirmative and the negative can be defended with equally valid proof and logically correct arguments. But these four double statements present each two contradictory affirmations of which only one side can be right; or if both are right they must be affirmed according to the sense which we attach to the words or as we interpret the meaning of the proposition. Antinomies are contradictory, and, according to Kant, they express a deeper truth than reason can fathom, the word being derived from *anti* (in the sense of "contrary") and *nomos* (law).

Reason, according to Kant, is not absolute. Reason is the order which dominates the world of reality, and this world of reality is conditioned. Why the world is thus conditioned is another question, but where the world touches its boundaries in time and space we are confronted with different conditions and reason is no longer our reliable guide. Here nature changes into metaphysics, and so it is to be expected that Kant's problem here touches the religious problems of philosophy.

Kant's four antinomies is the field of theology, but we must grant that they have not been taken seriously by naturalists, who, as a rule, have ignored them. In the hands of thinking theologians however they have become a welcome weapon in favor of an ultimate agnosticism and have proved the foundation for the acceptance, by rationalistic adherents of Kant, of church dogmas which otherwise would have proved objectionable.

The first antinomy concerns the problem of infinity, and here Kant takes infinity as an object, a thing that exists in a concrete bodily shape. Naturally he is puzzled, and, while knowing that infinity is a clear and undeniable thought in mathematics, he is unable to point it out in the world of reality. If Kant had investigated the nature of infinity he would have come to the conclusion that it is not a concrete object but a function. Infinity is a possibility, not a thing. All things are bodies of a definite mass, with definite limits, but infinity is unlimited; it is a process that can be carried on without end.

Therefore infinity is a feature of certain actions or processes, but it is never an actual event. For instance I can count to one thousand and farther; I can count to one million and farther; there is no limit. I can count to a billion, and still there is no end of counting. I can think a series of numbers amounting to infinitude, and even then I need not stop, for I can invent transfinite numbers, calling them $\infty+1$, $\infty+2$, $\infty+3$, etc. Infinitude is always a function, a possibility; and whenever I have a real concrete number it is not infinite. A real count of an actual distance, be it in space, as in miles, or in time, as in years, may be ever so great but it is always definite and finite.

Infinitude is not a problem too deep for reason, nor is it an unmeaning idea. Mathematics has made good use of it, and theology employs the term to express functions that will not, or should not, be limited or clearly defined—for we should have a liberal margin for possibilities which we cannot as yet understand. In this

sense we call God infinite, to indicate that we can never grasp the full meaning of the God idea. But infinitude is not identical with divinity, for everything is infinite in its possibilities. The human soul is infinite and human progress is infinite; yea, every atom, every speck of actual existence is infinite. Every point in space is surrounded with infinitude, and the same may be said of every moment of time, the infinitude of time being called eternity. We can move in thought out into the world of space and reach Sirius, the nearest fixed star; we can go beyond and reach more distant stars; we can travel to the outskirts of the Milky Way and even then we have not reached the end of our journey. We can go farther and discover other stellar systems, on and on, leaving an infinitude behind and ever having an infinitude before us. And let us assume that at last we reach the outer limits of concrete existence but can still proceed into the surrounding void. This means that while the concrete immensity of the world must be definite and at a definite time, space itself, which is not a concrete thing but the possibility of existence or the scope of motion (as we have defined it elsewhere), is infinite.

And the same is true of time. We may go back to the moment when our solar system originated from the conflagration of the world-dust that was gathered in the mighty whirls of a collision of mighty comets. We might go further back to the time when these comets were worlds, as is now our solar system; and beyond to the time when they originated. We might go back still further, when the whole Milky Way differentiated into clusters of commotion in an ocean of indifferent ether. Here we must stop for we know nothing of ether, as we may call the primordial stuff which by contraction and differentiation is capable of forming matter in its successive stages as chemical elements. But suppose that, previous to the original contraction of the primordial ether, there has been a calm of non-activity, we can still further go back into emptiness. Just as we count now from the birth of Christ as *A.D.* and the time before Christ as *B.C.*, so we could speak of world-eons since the origin of the cosmos and of world-eons before the birth of the cosmos. Eternities stretch out before us as well as behind us. Time is infinite.

Besides this conception of infinitude and eternality there are the aspects of relations which are independent of time and space. Such are natural laws, or mathematical theorems, or universal principles of thought, or fundamental principles of right, of truth,

of morality. They are also called eternal, but this eternality is different from the eternality of continued existence. This quality of being above time and space denotes the universality of certain relations; and these indicate their independence of localized and temporary realization and bear the stamp of intrinsic or absolute necessity.

When we understand the nature of infinitude we see at once that every concrete thing and every definite event, also the definite entirety of our solar system, yes even the immeasurable cosmos of the Milky Way with its uncounted stars, has a definite beginning and a definite end. Nothing concrete and particular is infinite. Infinitude is a process, a function, a thought or a plan or a possibility of unending continuance.

The laws of existence, those uniformities which condition the formation of all things, are independent of space and time; they are infinite and eternal in the sense that they are superspatial and supertemporal. They are intrinsically necessary and constitute that which is commonly called Deity or God. They are the formative factors of existence and in religion have been personified as Creator.

Such considerations throw light upon the nature of all the four antinomies of Kant. Here are the four antinomies stated in non-contradictory terms:

1. The actual and concrete universe is temporal and limited, but existence in its potentialities is infinite and the conditions of existence are eternal and infinite. The norms or laws, called God in religion, are superspatial and supertemporal.

2. As a mathematical line is divisible into infinitely small portions, so every atom is theoretically possessed of parts, of an upper and lower portion, a right and left side, an inside and an outside, etc. It is possible that our chemists cannot produce the conditions under which we shall be able to resolve the atoms, but that is not the question in Kant's antinomies. Theoretically every concrete body, even the smallest one, be it an atom or a molecule, is of a definite size and consists of definite parts. We may assume that every molecule has originated under definite conditions, by a law of formation still unknown to us, from a primitive homogeneous stuff, perhaps the ether, but we must assume that every one of the tiniest units of actual mass represents a definite amount of ether and that these minute portions themselves are also in turn theoretically infinitely divisible, as is the smallest geometrical line.

3. In connection with the third antinomy, I refer to my dis-

cussions of free will in previous expositions of the problem. The problem of free will is based upon a confusion of the words necessity and violence or compulsion. I act freely, that is, according to my free will, if my action is determined by my own character, if it expresses my own true will and if I act without compulsion. If I am compelled to act against my will I am not free but suffer violence.

In either case, whether I be free in my decision or compelled to act against my will, my action is strictly determined. If my free action were undetermined, free will would be haphazard and without moral significance. Thus causality is a universal law which dominates even the decisions of free will, and the theory of indeterminism is simply a confusion of thought.

4. Finally, the question of God, which Kant bears in mind when discussing the antinomy of an absolutely necessary being, resolves itself into the meaning of necessity. Certainly there is not an absolutely necessary being, but there is necessity. This necessity is not a being, not a creature, not an individual, not a person, not anything concrete, not a somebody who is in a definite space and is eternal in the sense of an enduring or continued temporal existence. Necessity is a factor, a norm, a regulative principle. It is supertemporal and superspatial. It possesses all the qualities which philosophical thought has attributed to God. It is the creator, for it is the formative principle of the world. It is the ruler and Lord of the universe, for it is the law, the totality of all natural laws, physical, psychical and moral. It is omnipresent; it is eternal; it is unfailing; it is the standard of both truth and error, right and wrong. This conception of God is not personal, but superpersonal; it is the condition under and through which, in the course of evolution, the rational, i. e., human personality is produced.

The antinomies of Kant have done their service. Religious people became convinced that the ideas of God, freedom and immortality had become untenable in the modern interpretation of dogmatic Christianity. And yet they felt that they contained great truths which could not be dispensed with. Thus the Kantian notion that they were *transcendent,* that they were profounder than human comprehension, was a welcome principle which found an apparently rational basis in the theory of the four antinomies.

From the standpoint of a rigid radicalism such as I advocate the contradictory form of the antinomies can be disregarded and their meaning stated in clear terms; or it can be preserved if its

words are interpreted in two different senses. And antinomies contain a truth but no real contradiction.[1]

The truth underlying the antinomies is preserved by the higher conception of God, the doctrine of the Overgod, which I preach, and by clearness of thought as to the meaning of infinitude and an elucidation of the true nature of free will.

EDITOR.

PROFESSOR VON GARBE ON THE JEWS OF MALABAR.

In the April number of *The Monist* (p. 283) Mr. Wilfred H. Schoff asserts the probability of the tradition of the Jews of Malabar which claims that they had migrated to India in 68 A. D., to the number of 10,000, although he doubts whether there were so many of them. This tradition, however, is not "well attested" as Schoff says, but originated in the late Middle Ages. Also the "ancient" Cochin grant of Bhâskara Revivarman quoted by Fleet from the *Epigr. Ind.*, III, 66, is a late copper-plate grant in the Tamil language.

But to pass judgment on such things one must constantly bear in mind how important it is to regard all learned traditions (especially late ones) of the Orient with a critical eye.

Benjamin of Tudela only went as far as Persia in the second half of the twelfth century; and although it may seem quite probable from his report and that of Marco Polo (end of the thirteenth century) that there were Jews in India in *their* time, this proves nothing for a thousand years earlier. Without *trustworthy* sources for the antiquity of Judaism in Malabar no definite conclusion can be reached.

The *possibility* that Jews may have come to Malabar at quite an early date is not, to be sure, excluded; but if they did come, it was not in large numbers but singly. In Abyssinia also, where the native Jews are black and speak an Agau language, and in Yemen, certainly only comparatively few have spread Judaism among the natives, just as in the case of the Christians. But Malabar is pretty distant from the starting-point of Jewish migration, and the Jews have never been a sea-faring people except once, and that was in Solomon's time under the protection of the Phenicians who served

[1] The same subject has been discussed in my treatment of Kant's philosophy in connection with my translation of his *Prolegomena*.

as their guides. In order to judge the case in question, the analogy of the Thomas Christians is instructive. They were not baptized by an apostle as their own tradition recorded, but the sect originated at a much later time.

The literature on the white and black Jews in Malabar might be considerably multiplied, but without throwing any light on the historicity of the native tradition.

TÜBINGEN, GERMANY. R. GARBE.

MATHEMATICIANS AND PHILOSOPHERS.

Jonathan Swift, in the second chapter of that part of *Gulliver's Travels* which describes Gulliver's third voyage, made Gulliver say of the mathematicians of Laputa:

"They are very bad reasoners and vehemently given to opposition unless when they happen to be of the right opinion, which is seldom their case. Imagination, fancy and invention they are wholly strangers to, nor have they any words in their language by which those ideas can be expressed; the whole compass of their thoughts and mind being shut up within the two fore-mentioned sciences [mathematics and music].

"Most of them, and especially those who deal in the astronomical part, have great faith in judicial astrology although they are ashamed to own it publicly. But what I chiefly admired, and thought altogether unaccountable, was the strong disposition I observed in them towards news and politics, perpetually inquiring into public affairs, giving their judgments in matters of state, and passionately disputing every inch of a party opinion. I have indeed observed the same disposition among most of the mathematicians I have known in Europe, although I could never discover the least analogy between the two sciences."

Gulliver's Travels was published in 1726. In 1734 George Berkeley (1685-1753), the famous philosopher and Bishop of Cloyne in Ireland, published *The Analyst, or a Discourse to an Infidel Mathematician,*[1] the object of which was to show that the principles of the infinitesimal calculus are no clearer than, or perhaps not as clear as, the principles of Christianity. The "infidel mathe-

[1] Cf. on this book and the controversy to which it gave rise M. Cantor, *Vorlesungen über Geschichte der Mathematik,* Vol. III, 2d. ed., Leipsic, 1901, pp. 737-746; Cf. Vol. IV (Leipsic, 1908; article by G. Vivanti, pp. 644, 648, 649) for references to Berkeley on the subject of the theory of the compensation of errors with Lagrange and Lazare Carnot.

matician" referred to was Edmund Halley (1656-1742), the distinguished astronomer and friend of Isaac Newton, and of whom the story is told[2] that, when he indulged in jest concerning theological questions, he was curtly repulsed by Newton with the remark: "I have studied these things; you have not!"

The incident which led Berkeley to write his *Analyst* was this. A friend of Berkeley's refused the offer of spiritual consolation when he was on a bed of sickness because Halley, the skilled mathematician, had convinced him of the inconceivability of the doctrines of Christianity. Now Berkeley showed very skilfully that the principles of infinitesimal analysis—the method of fluxions, in Newton's terminology—were by no means clear. The *Analyst* provoked a great deal of controversy: Dr. James Jurin wrote under a pseudonym *Geometry No Friend to Infidelity*; a Dublin professor named Walton wrote a *Vindication of Sir Isaac Newton's Principles of Fluxions*; Berkeley replied with a *Defence of Freethinking in Mathematics*; Jurin wrote in 1735 *The Minute Mathematician, or the Freethinker no Just Thinker*;[3] Benjamin Robins and Pemberton entered the lists on behalf of Newton's method of fluxions and against Jurin's clumsy defence of it; and lastly, Berkeley's *Analyst* has the credit of inspiring Maclaurin to write his famous *Treatise of Fuxions* of 1742, in which a rigorous foundation of the method of fluxions was attempted.

"The *Analyst*," said De Morgan,* "was intentionally a publication involving the principle of Dr. Whately's argument against the existence of Buonaparte; and Berkeley was strictly to take what he found. The *Analyst* is a tract which could not have been written except by a person who knew how to answer it. But it is singular that Berkeley, though he makes his fictitious character nearly as clear as afterwards did Whately, has generally been treated as a real opponent of fluxions. Let us hope that the arch Archbishop [Whately] will fare better than the arch Bishop."

But Berkeley's tract had another merit. In it are given the foundations of the theory that the correct results of the infinitesimal calculus are obtained by a compensation of errors. This theory

[2] Mach, *Mechanics* (Chicago, 1907), pp. 448-449.

[3] The most comical mis-translation I have ever come across is of the title of a work of which this reminds me, I mean Berkeley's *Alciphron, or the Minute Philosopher* (1733)—a dialogue in which he critically examines the various forms of freethinking in the age. A "minute philosopher" is, of course, a philosopher who examines things minutely; but Montucla, in his *Histoire des Mathématiques*, translates it as "le petit philosophe"!

* *Phil. Mag.*, Nov., 1852.

was rediscovered, apparently independently of others and of each other, by Lagrange and Lazare Carnot in 1797.

In view of this merit, R. Adamson[4] seems rather too severe. Speaking of another work of Berkeley's he says that a "great part of the *Common Place Book* [containing Berkeley's thoughts on physics and philosophy from about 1703] is occupied with a vigorous and in many points exceedingly ignorant polemic against the fundamental conception of the fluxional and infinitesimal calculus, a polemic which Berkeley carried on to the end of his days." Also it is hardly correct to say, with Adamson: "....in his *Analyst* he attacked the higher mathematics, as leading to freethinking; this involved him in a hot controversy." Berkeley did not attack mathematics; only the vague ideas and expositions of some mathematicians.

If the fact that Berkeley's early mathematical work was bad is any excuse for depreciating his later mathematical work, then indeed the *Analyst* might excusably be condemned; for Berkeley's early mathematical work of 1707 was, according to Cantor,[5] insignificant. But the theory of the compensation of errors in the infinitesimal calculus is perfectly correct and was, apparently,[6] stated for the first time by Berkeley.

* * *

Most mathematicians, like the unphilosophical among men of science, are so occupied with the use of methods which, judged by the results of years of application to the problems of nature, are manifestly reliable, that they too often succumb to the temptation of taking d'Alembert's maxim for mathematicians—*Allez en avant, la foi vous viendra*—as a maxim for logicians and philosophers, and of treating with hasty contempt the criticisms of philosophers. In the case of Berkeley, the mathematicians missed the point as much as Dr. Johnson did when he refuted Berkeleyanism by kicking a stone. In the case of Hegel, the mathematicians appear to have been more in the right. The principal criticism to which Hegel's criticism is subject is that it is too uncritical: it accepts some mathematician's obscurities and concludes obscurity in mathematics. The mathematicians denied obscurity in mathematics, and they were right. But still the fundamental conceptions

[4] *Encycl. Brit.*, 9th ed., Vol. III, 1875, p. 590.

[5] *Op. cit.*, Vol. III, p. 737.

[6] Berkeley's opponents, apparently mistakenly, said that the idea of compensation was old (*ibid.*, pp. 743-744).

of the infinitesimal calculus had not been presented in a logically rigorous form when Hegel wrote.

The care for exactness in dealing with principles is of comparative late growth in mathematics. We shall not be far wrong if we put its birth after Kant published his great *Critique* in 1781. I cannot find any evidence for a direct influence of Kant on Lagrange, Gauss, Cauchy, or Weierstrass; it seems that criticism was "in the air." And so, in the settlement of the logical and philosophical difficulties of mathematics philosophers have not hitherto had a large share. ".... Philosophy asks of Mathematics: What does it mean? Mathematics in the past was unable to answer, and Philosophy answered by introducing the totally irrelevant notion of mind. But now Mathematics is able to answer, so far at least as to reduce the whole of its propositions to certain fundamental notions of logic. At this point, the discussion must be resumed by Philosophy."[7]

* * *

There is another aspect of the distinction between mathematicians and logicians. In modern times, from the time of Leibniz up to the middle of the nineteenth century, the only mathematicians of eminence who were also eminent logicians were John Wallis (1616-1703) and perhaps Leonhard Euler (1707-1783). About the middle of the nineteenth century there began, of course, with Boole and De Morgan, a new era for logic, in which the symbolism and methods of algebra were used to give generality and precision to logical conclusions and to create new logical methods. "Every science," says De Morgan,[8] "that has *thriven* has thriven upon its own symbols: logic, the only science which is admitted to have made no improvements in century after century, is the only one which has *grown no symbols.*" Again, De Morgan in his *Syllabus,*[9] says: "I end with a word on the new symbols which I have employed. Most writers on logic strongly object to all symbols except the venerable *Barbara, Celarent,* etc..... I should advise the reader not to make up his mind on this point until he has well weighed two facts which nobody disputes, both separately and in connection. Firstly, logic is the only science which has made no progress since the revival of letters; secondly, logic is the only science which has produced no growth of symbols."

[7] B. Russell, *The Principles of Mathematics,* Cambridge, 1903, p. 4; cf. pp. 129-130.

[8] *Trans. Camb. Phil. Soc.,* Vol. X, 1864, p. 184.

[9] *Syllabus of a Proposed System of Logic,* London, 1860, p. 72.

But De Morgan saw the advantages that would result from the use by logic of a symbolism analogous to the algebraical. In his third paper "On the Syllogism"[10] he says: "As joint attention to logic and mathematics increases, a logic will grow up among the mathematicians, distinguished from the logic of the logicians by having the mathematical elements properly subordinated to the rest. This 'mathematical logic'—so-called *quasi lucus a non nimis lucendo*—will commend itself to the educated world by showing an actual representation of their form of thought—a representation, the truth of which they recognize—instead of a mutilated and one-sided fragment, founded upon canons of which they neither feel the force nor see the utility."

* * *

At the present time the prejudice of logicians against the use of symbols that happen to have been used beforehand in mathematics has almost disappeared, thanks principally to the work of Dr. J. Venn.[11] The whole objection of the old-fashioned logicians really rested on no better grounds than this: The use of x and y, which are used in mathematics, ought not to be used instead of the logical X and Y, *because x and y have been used for something quantitative.*[12] As if the word "cabbage" had any rigid connection with the essence of the vegetable of which that word reminds us! In symbolic logic the arithmetical signs +, −, and × were used because certain logical operations have many analogies with arithmetical operations. There is no objection that can be urged against this—a fertile source of discoveries—*except* that when we write out mathematical theorems in symbolic logic there may be a confusion of terms. But we must be careful not to pursue the analogy too far. Logical addition, for example, and mathematical addition are not identical. There is a point where the analogy breaks down. And when we go deeply into the matter, the differences will begin to outweigh the identities in importance. Broadly speaking, we may say that *modern logic is symbolic but has got beyond the rather evident analogies it has with algebra,* and a man who seemed rather deep and abstract in his mathematics and logic fifty years ago now seems rather a superficial and *naïf* person. In one form or another, analogy probably is always guiding us in our researches, but, as we

[10] *Trans. Camb. Phil. Soc.,* Vol. X, 1864, note on page 176.
[11] *Symbolic Logic,* London, 1881, 2d ed. 1894.
[12] Cf. *ibid.,* p. ix (of either edition).

progress in subtlety, we become more and more convinced of the limitations of the more obvious analogies.

* * *

Let us now return to Swift. In his description of Gulliver's voyage to Laputa, he describes the mathematicians of that country as silly and useless dreamers, whose attention has to be awakened by flappers. Also, the mathematical tailor measures his height by a quadrant, and deduces his other dimensions by a rule and compasses, producing a suit of very illfitting clothes. On the other hand, the mathematicians of Laputa, by their marvelous invention of the magnetic island floating in the air, ruled the country and maintained their ascendency over their subjects. Dr. Whitehead[13] says: "Swift, indeed, lived at a time peculiarly unsuited for gibes at contemporary mathematicians. Newton's *Principia* had just been written, one of the great forces which have transformed the modern world. Swift might just as well have laughed at an earthquake." We cannot wholly subscribe to this, for it seems not unlikely that Swift, like everybody else, could not doubt the usefulness, importance, and correctnesss of the mathematician's work, but shared, with the philosopher, a doubt of the mathematician's being able to state his principles clearly and reasonably, just as we may doubt the existence of a knowledge of thermodynamics in a man who drives a railway engine.

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CURRENT PERIODICALS.

In the number of *Scientia* for August, 1915, Georges Bohn gives the second part of his article on new ideas on adaptation and evolution. It is interesting to notice that, according to the author, both Lamarck and Darwin were finalists. E. Carnevale contributes the second part of his study on democracy and penal justice. The articles concerned with questions raised by the war are by W. J. Ashley on "The Economic Conversion of England" and Charles Guignebert on the part played by the Roman Catholic Church— or what, according to him is the same thing, the Pope—in the European war. There is a short note by Federigo Enriques on the art of writing a treatise, prompted by his forthcoming book on the geometrical theory of equations and algebraic functions. There are reviews of books and periodicals, and French translations of the Italian and English articles.

[13] A. N. Whitehead, *An Introduction to Mathematics*, London, 1911, p. 10.

It is very pleasant to see the *Revue de Métaphysique et de Morale,* whose publication has been suspended since the beginning of the European war. The number for September, 1914, appeared in June, 1915, and the only mark of the war on it is the article by Gustave Belot on war and democracy, and a study by André Lalande of the work of Louis Couturat. Couturat was a victim of the war: though not a combatant, he was killed on the day (August 3, 1914) on which Germany declared war on France, by a heavy automobile which was carrying orders of mobilization. The article is followed by a bibliography of Couturat's works together with a list of some of the reviews and translations of them. Couturat's literary activity may broadly be characterized by the names of his principal publications: (1) *De l'infini mathématique* of 1896; (2) *La Logique de Leibniz* of 1901 and *Opuscules et fragments inédits de Leibniz* of 1903; (3) *L'Algèbre de la Logique* of 1905; (4) *Les Principes des Mathématiques* of 1905. Since 1901 Couturat has been more and more busied with the problem of an international language, and Ido in particular, and here we may remind our readers of his article in *The Monist* for 1905. The other articles in this number of the *Revue* are Emile Boutroux's presidential address (April, 1914) to the international congress of mathematical philosophy; C. Bouglé's remarks on "polytelism"—the multiplicity of ends that one and the same means allows us to reach—; Désiré Roustan's on science as a vital instrument, and Léon Cahen's publication of some hitherto unpublished fragments of Condorcet on instinct, the words "nature" and "natural," and so on.

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Scientia (*Rivista di Scienza*) begins, with the first number (June and July, 1915) of the eighteenth volume, a second series, under the sole editorship of Eugenio Rignano. The reason for this is that in the future there will be, in addition to articles on scientific synthesis and organization, articles of international interest which carry a political responsibility that cannot be divided among a board of editors. The inquiry upon the causes of the war ends in this number, but there will be in future discussions of various problems raised by the war. The only purely scientific article in this number is one by Georges Bohn, pointing out that there are disharmonies in living beings, and that biologists, like metaphysicians, have hitherto had recourse to a theory of adaptation for special ends to explain what they took to be harmonies. The inquiry upon the war consists of a French article by Louis Havet, an English article by J. Holland Rose emphasizing the part played by nationality, and an Italian summary of the whole inquiry by Eugenio Rignano. Besides this there are reviews of books and other periodicals and French translations of the English and Italian articles.

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The first article in *Science Progress* for July 1015 is by H. Spencer Jones, "On the Structure of the Universe." The many re-

cent researches upon the distances and distribution of stars, upon the relative distances of stars of different types, upon the numbers of stars of different magnitudes, and upon allied topics, have all contributed in throwing some light on the problem of the structure and the evolution of the universe; and the author gives an account of these researches. The second article is an answer to Miss Stebbing's criticisms on Mercier's logic, by Charles A. Mercier. "Miss Stebbing's reply to my charges against logic," says the author, "does not seem to me successful, but as it does seem successful to some people, a rejoinder may be permitted." It is as impossible for a modern logician to deny the essential justice of Mercier's attack on the futile stuff taught as "logic" in the schools and the antiquated doctrine that the syllogism is the only principle of reasoning, as it is for any one with a sense of humor not to be amused by his writings. Maurice Copisarow gives a technical paper on "Carbon: Its Molecular Structure and Mode of Oxidation." None of the theories of Lang (1888), Baker (1888) and Dixon (1896, 1899), and Rhead and Wheeler (1910-1913) are absolutely wrong or a complete representation of the facts. The author starts from three fundamental assumptions as a basis. "i. A carbon molecule is polyatomic (This is suggested by its high volatilization-point and products of moist oxidation). ii. A carbon atom is potentially always tetravalent (Comberg's work on triphenylmethyl and Nef's on polymethylene compounds do not necessarily imply the non-tetravalency of a carbon atom). iii. Carbon exists in three allotropic modifications (Several new modifications suggested by Brodie, Berthelot, Luzi, and others have been proved by Moissan and Le Chatelier to be either compounds or solutions and mixtures of carbon with some other element)." There is an imaginative blank verse by George William Bettany on "A Bit of Rock." D. F. Harris and H. J. M. Creighton write on "The Role of Reductase in Tissue Respiration,"—also a technical paper. S. C. Bradford writes on "The History of Adrenalin." "The story of the discovery of the function of the suprarenal capsules, followed by the isolation of the active principle of their secretion, the determination of its structure, and its subsequent synthesis, forms one of the most fascinating chapters in the history of bio-chemistry." Of more general interest is the discussion by A. G. Thacker of "Some Eugenic Aspects of War." W. Lawrence Balls contributes an interesting account of "The Spinning Properties of Cotton." In a recent number of *Science Progress* he has indicated some of the ways by which purely scientific investigations were likely to yield results of economic value to the cotton trade. The present paper is to show how some unexpected light has since been thrown upon the causes on which the strength of yarn depends, thereby indicating the possibility of a substantial advance in the technique of spinning. Besides these articles, the number contains interesting essay-reviews and "Recent Advances in Science: Mathematics, Astronomy, Physics, Chemistry, Geology, Botany, Zoology, Anthropology." There are also notes and short reviews of various books. Φ